The International Association for the Properties of Water and Steam

Santorini, Greece
July 2005

Supplementary Release on Backward Equations for Specific Volume as a Function of Pressure and Temperature $v(p,T)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam

President:
Professor F. Marsik
Institute of Thermomechanics
Academy of Sciences of the Czech Republic
Dolejskova 5
18200 Prague 8, Czech Republic

Executive Secretary:
Dr. R. B. Dooley
Electric Power Research Institute
1300 West W. T. Harris Blvd.
Charlotte, North Carolina 28262, USA

This release contains 35 pages, including this cover page.

This supplementary release has been authorized by the International Association for the Properties of Water and Steam (IAPWS) at its meeting in Santorini, Greece, 3-8 July, 2005, for issue by its Secretariat. The members of IAPWS are: Argentina and Brazil, Britain and Ireland, Canada, the Czech Republic, Denmark, France, Germany, Greece, Italy, Japan, Russia, and the United States of America.

The backward equations $v(p,T)$ for Region 3 provided in this release are recommended as a supplement to "The IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [1, 2] and to the Supplementary Releases on "Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ for Regions 1 and 2" (referred to here as IAPWS-IF97-S01) [3, 4], "Backward Equations for the Functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for Region 3" (referred to here as IAPWS-IF97-S03rev) [5, 6], and "Backward Equations $p(h,s)$ for Region 3, Equations as a Function of h and s for the Region Boundaries, and an Equation $T_{\text{sat}}(h,s)$ for Region 4" (referred to here as IAPWS-IF97-S04) [7, 8]. Further details concerning the equations of this supplementary release can be found in the corresponding article by H.-J. Kretzschmar *et al.* [9].

Further information concerning this supplementary release, IAPWS-IF97, IAPWS-IF97-S01, IAPWS-IF97-S03rev, IAPWS-IF97-S04, and other releases issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from http://www.iapws.org.

Contents

1 Nomenclature

Thermodynamic quantities:

c_p Specific isobaric heat capacity
f Specific Helmholtz free energy
h Specific enthalpy
p Pressure
s Specific entropy
T Absolute temperature [a]
v Specific volume
w Speed of sound
θ Reduced temperature $\theta = T/T^*$
π Reduced pressure, $\pi = p/p^*$
ω Reduced volume, $\omega = v/v^*$
Δ Difference in any quantity

Superscripts:

97 Quantity or equation of IAPWS-IF97
01 Equation of IAPWS-IF97-S01
03 Equation of IAPWS-IF97-S03rev
04 Equation of IAPWS-IF97-S04

* Reducing quantity
' Saturated liquid state
" Saturated vapor state

Subscripts:

1…5	Region 1…5
3a …3z	Subregion 3a…3z
3ab	Boundary between subregions 3a, 3d and 3b, 3e
3cd	Boundary between subregions 3c and 3d, 3g, 3l, 3q, 3s
3ef	Boundary between subregions 3e, 3h, 3n and 3f, 3i, 3o
3gh	Boundary between subregions 3g, 3l and 3h, 3m
3ij	Boundary between subregions 3i, 3p and 3j
3jk	Boundary between subregions 3j, 3r and 3k
3mn	Boundary between subregions 3m and 3n
3op	Boundary between subregions 3o and 3p
3qu	Boundary between of subregion 3q and 3u
3rx	Boundary between of subregion 3r and 3x
3uv	Boundary between subregions 3u and 3v
3wx	Boundary between subregions 3w and 3x
B23	Boundary between regions 2 and 3
c	Critical point
it	Iterated quantity
max	Maximum value of a quantity
RMS	Root-mean-square value of a quantity
sat	Saturation state
tol	Tolerated value of a quantity

Root-mean-square value:

$$\Delta x_{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\Delta x_n)^2}$$

where Δx_n can be either absolute or percentage difference between the corresponding quantities x; N is the number of Δx_n values (10 million points uniformly distributed over the range of validity in the p-T plane).

[a] Note: T denotes absolute temperature on the International Temperature Scale of 1990 (ITS-90).

2 Background

The IAPWS Industrial Formulation 1997 for the thermodynamic properties of water and steam (IAPWS-IF97) [1, 2] contains basic equations, saturation equations and equations for the frequently used backward functions $T(p,h)$ and $T(p,s)$ valid in the liquid region 1 and the vapor region 2; see Figure 1. IAPWS-IF97 was supplemented by "Supplementary Release on Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [3, 4], which will be referred to as IAPWS-IF97-S01. These equations are valid in region 1 and region 2. An additional "Supplementary Release on Backward Equations for the Functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [5, 6], which will be referred to as IAPWS-IF97-S03rev, was adopted by IAPWS in 2003 and revised in 2004. In 2004, IAPWS-IF97 was supplemented by "Supplementary Release on Backward Equations $p(h,s)$ for Region 3, Equations as a Function of h and s for the Region Boundaries, and an Equation $T_{\text{sat}}(h,s)$ for Region 4 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (referred to here as IAPWS-IF97-S04) [7, 8].

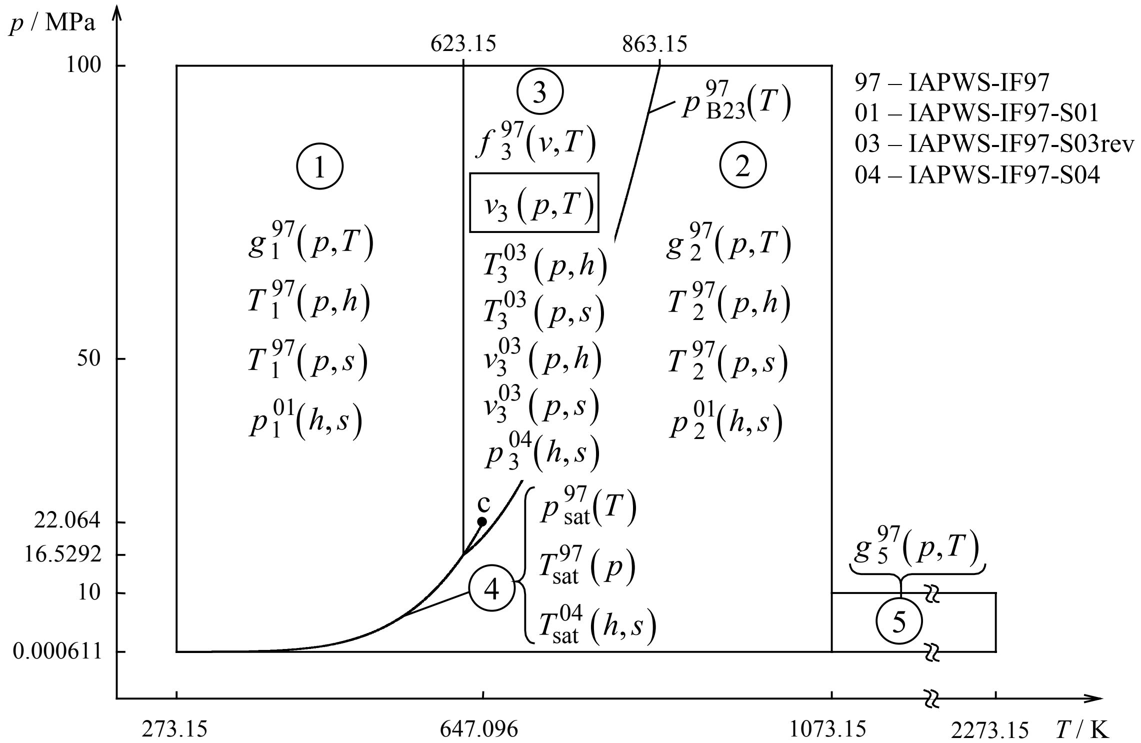

Figure 1. Regions and equations of IAPWS-IF97, IAPWS-IF97-S01, IAPWS-IF97-S03rev, IAPWS-IF97-S04, and the equations $v_3(p,T)$ of this release

IAPWS-IF97 region 3 is covered by a basic equation for the Helmholtz free energy $f(v,T)$. All thermodynamic properties can be derived from the basic equation as a function of specific volume v and temperature T. However, in modeling some steam power cycles, thermodynamic properties as functions of the variables (p,T) are required in region 3. It is cumbersome to perform these calculations with IAPWS-IF97, because they require iterations of v from p and T using the function $p(v,T)$ derived from the IAPWS-IF97 basic equation $f(v,T)$.

In order to avoid such iterations, this release provides equations $v_3(p,T)$; see Figure 1. With specific volume v calculated from the equations $v_3(p,T)$, the other properties in region 3 can be calculated using the IAPWS-IF97 basic equation $f(v,T)$.

For process calculations, the numerical consistency requirements for the equations $v(p,T)$ are very strict. Because the specific volume in the p-T plane has a complicated structure, including an infinite slope at the critical point, region 3 was divided into 26 subregions. The first 20 subregions and their associated backward equations, described in Section 5, cover almost all of region 3 and fully meet the consistency requirements. For a small area very near the critical point, it was not possible to meet the consistency requirements fully. This near-critical region is covered with reasonable consistency by six subregions with auxiliary equations, described in Section 6.

3 Numerical Consistency Requirements

The permissible value for the numerical consistency of the equations for specific volume with the IAPWS-IF97 fundamental equation was determined based on the required accuracy of the iteration otherwise used. The iteration accuracy depends on thermodynamic process calculations. To obtain specific enthalpy or entropy from pressure and temperature in region 3 with a maximum deviation of 0.001 % from IAPWS-IF97, and isobaric heat capacity or speed of sound with a maximum deviation of 0.01 %, a relative accuracy of $|\Delta v/v| = 0.001\,\%$ is sufficient. Therefore, the permissible relative tolerance for the equations $v(p,T)$ was set to $|\Delta v/v|_{\mathrm{tol}} = 0.001\,\%$.

4 Structure of the Equation Set

The range of validity of the equations $v_3(p,T)$ is region 3 defined by:

$$623.15\ \mathrm{K} < T \le 863.15\ \mathrm{K} \ \text{ and } \ p_{\mathrm{B23}}^{97}(T) < p \le 100\ \mathrm{MPa}.$$

The function $p_{\mathrm{B23}}^{97}(T)$ represents the B23-equation of IAPWS-IF97.

It proved to be infeasible to achieve the numerical consistency requirement of 0.001 % for $v_3(p,T)$ using simple functional forms in the region

$$T_{3\mathrm{qu}}(p) < T \le T_{3\mathrm{rx}}(p) \ \text{ for } \ p_{\mathrm{sat}}^{97}(643.15\ \mathrm{K}) < p \le 22.5\ \mathrm{MPa}\ ; \text{see Figure 2.}$$

This limitation is due to the infinite slope of the specific volume at the critical point. In order to cover region 3 completely, Section 6 contains auxiliary equations for this small region very close to the critical point.

Figure 2 shows the range of validity of the backward and auxiliary equations.

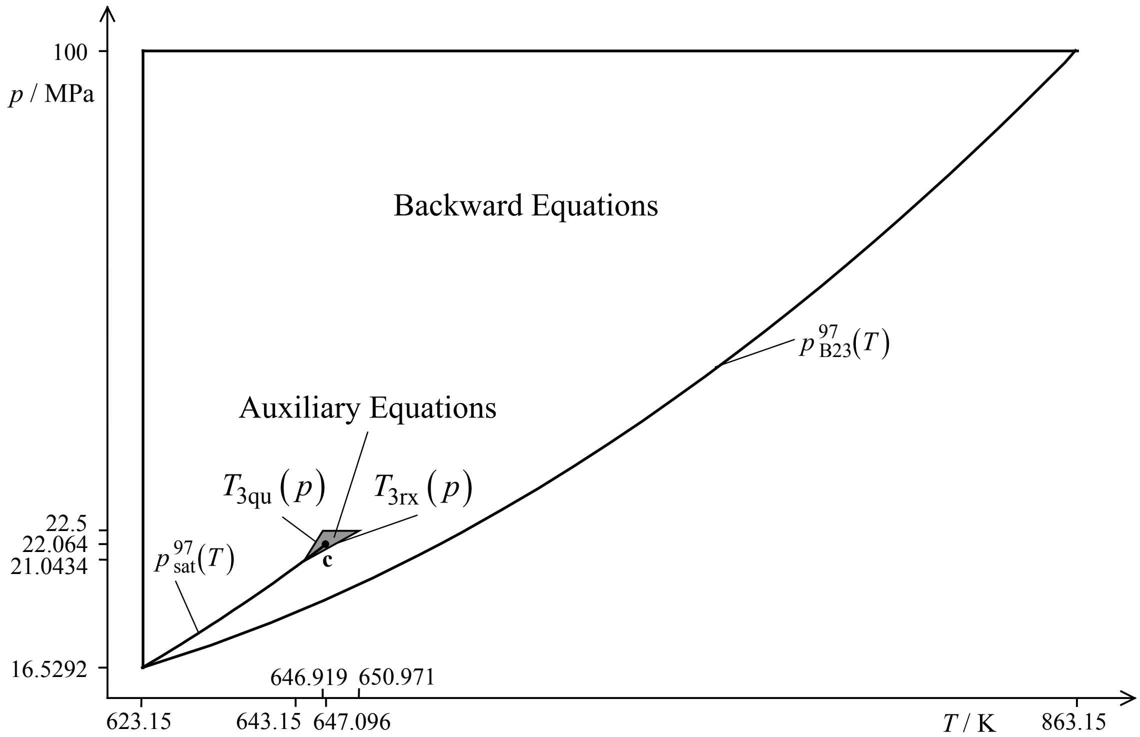

Figure 2. Range of validity of the backward and auxiliary equations. The area marked in grey is not true to scale but enlarged to make the small area better visible.

5 Backward Equations $v(p,T)$ for the Subregions 3a to 3t

5.1 Subregions

Preliminary investigations showed that it was not possible to meet the numerical consistency requirement with only a few $v(p,T)$ equations. Therefore, the main part of region 3 was divided into 20 subregions 3a to 3t; see Figures 3 and 4.

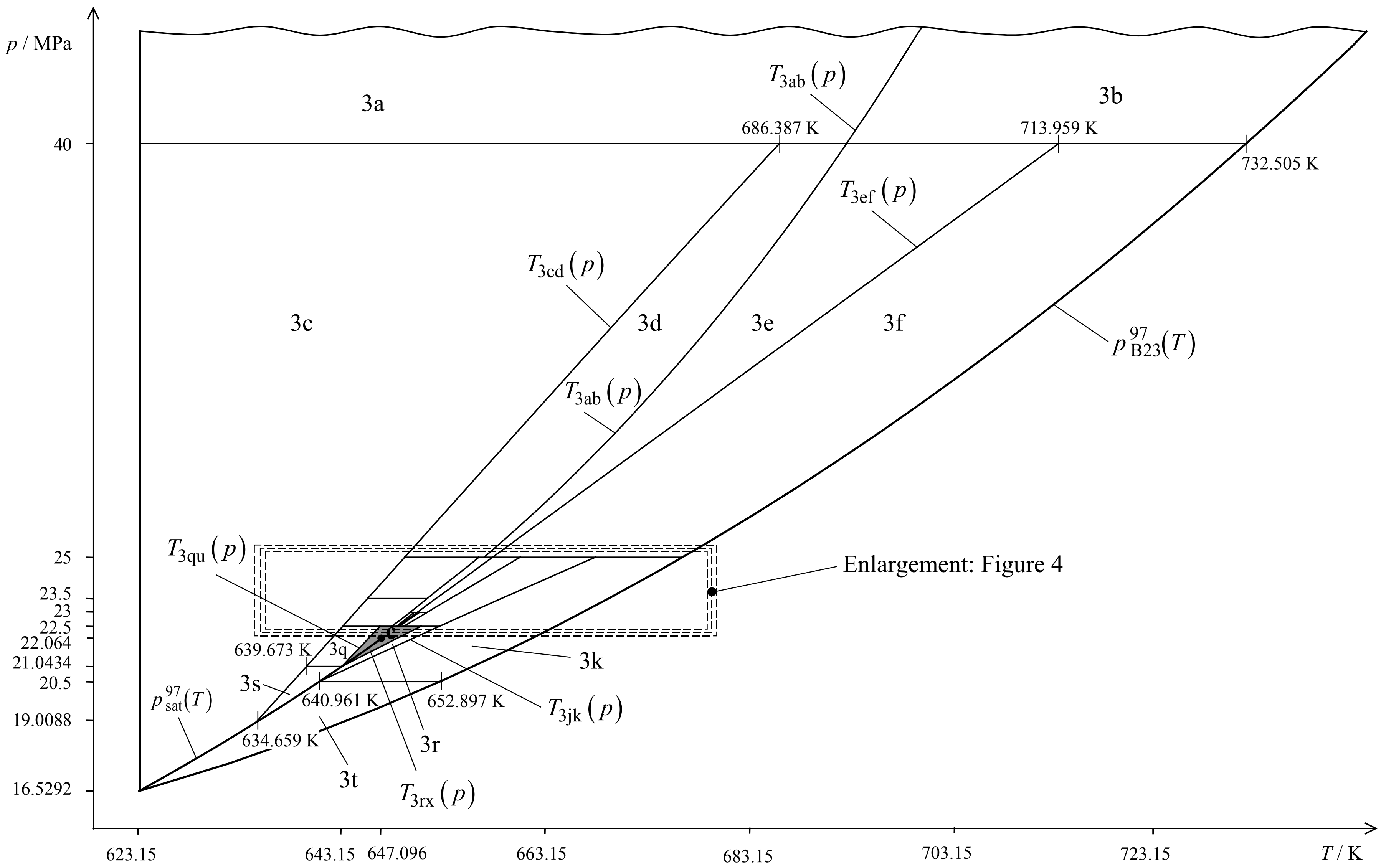

Figure 3. Division of region 3 into subregions for the backward equations $v_3(p,T)$

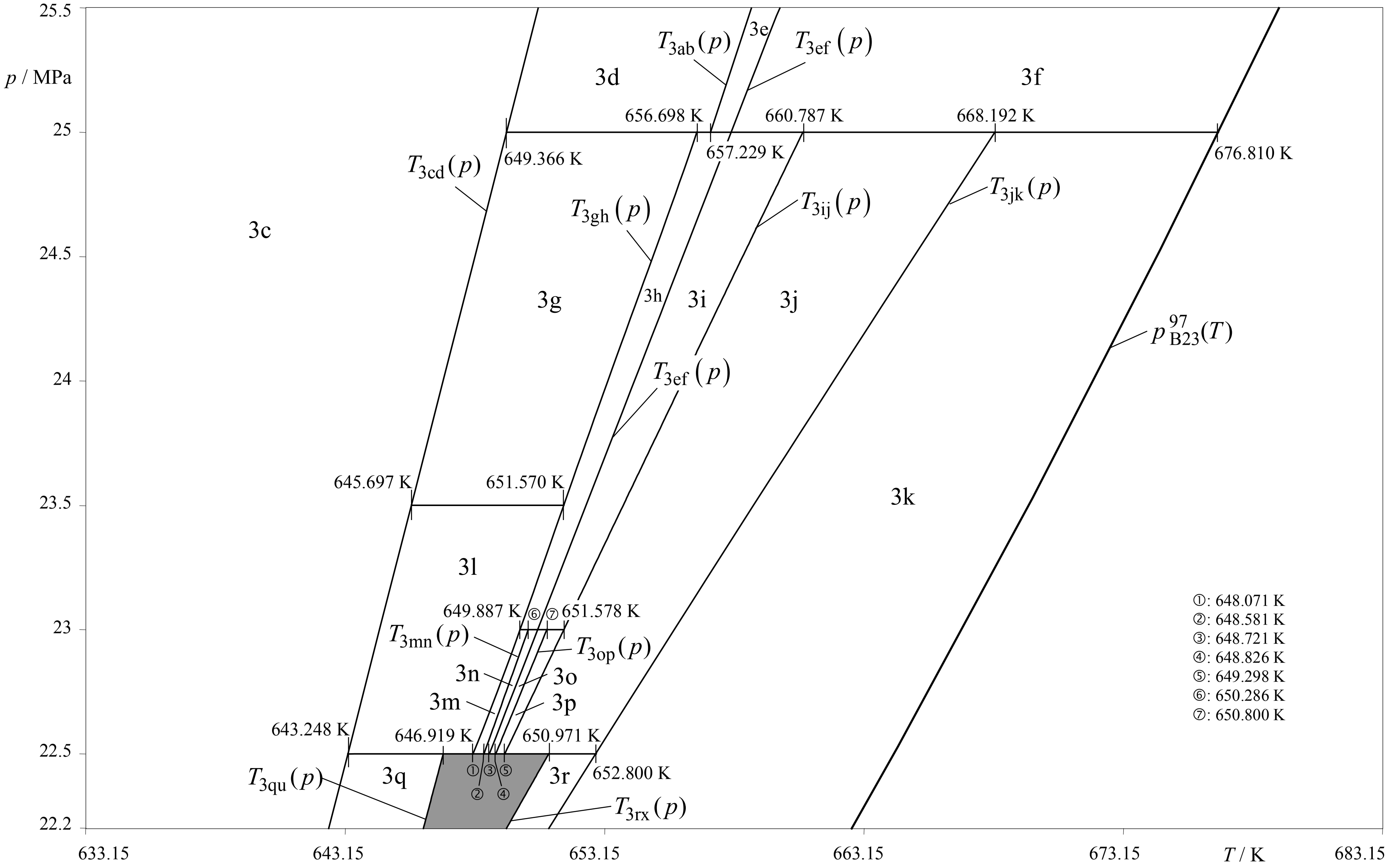

Figure 4. Enlargement from Figure 3 for the subregions 3c to 3r for the backward equation $v(p,T)$

The subregion boundary equations, except for $T_{3\text{ab}}(p)$, $T_{3\text{ef}}(p)$, and $T_{3\text{op}}(p)$, have the following dimensionless form:

$$\frac{T(p)}{T^*} = \theta(\pi) = \sum_{i=1}^{N} n_i \, \pi^{I_i} \ , \tag{1}$$

where $\theta = T/T^*$, $\pi = p/p^*$ with $T^* = 1\,\text{K}$, $p^* = 1\,\text{MPa}$.

The equations $T_{3\text{ab}}(p)$ and $T_{3\text{op}}(p)$ have the form:

$$\frac{T(p)}{T^*} = \theta(\pi) = \sum_{i=1}^{N} n_i \, (\ln \pi)^{I_i} \ , \tag{2}$$

and $T_{3\text{ef}}(p)$ has the form:

$$\frac{T_{3\text{ef}}(p)}{T^*} = \theta_{3\text{ef}}(\pi) = \left.\frac{\partial \theta_{\text{sat}}}{\partial \pi}\right|_{\text{c}} (\pi - 22.064) + 647.096 \ , \tag{3}$$

where $\partial\theta_{\text{sat}}/\partial\pi\big|_{\text{c}} = 3.727\,888\,004$.

The coefficients n_i and the exponents I_i of the boundary equations are listed in Table 1.

Table 1. Numerical values of the coefficients of the equations for subregion boundaries (except $T_{3\text{ef}}(p)$)

Equation	i	I_i	n_i	i	I_i	n_i
$T_{3\text{ab}}(p)$	1	0	$0.154\,793\,642\,129\,415 \times 10^{4}$	4	−1	$-0.191\,887\,498\,864\,292 \times 10^{4}$
	2	1	$-0.187\,661\,219\,490\,113 \times 10^{3}$	5	−2	$0.918\,419\,702\,359\,447 \times 10^{3}$
	3	2	$0.213\,144\,632\,222\,113 \times 10^{2}$			
$T_{3\text{cd}}(p)$	1	0	$0.585\,276\,966\,696\,349 \times 10^{3}$	3	2	$-0.127\,283\,549\,295\,878 \times 10^{-1}$
	2	1	$0.278\,233\,532\,206\,915 \times 10^{1}$	4	3	$0.159\,090\,746\,562\,729 \times 10^{-3}$
$T_{3\text{gh}}(p)$	1	0	$-0.249\,284\,240\,900\,418 \times 10^{5}$	4	3	$0.751\,608\,051\,114\,157 \times 10^{1}$
	2	1	$0.428\,143\,584\,791\,546 \times 10^{4}$	5	4	$-0.787\,105\,249\,910\,383 \times 10^{-1}$
	3	2	$-0.269\,029\,173\,140\,130 \times 10^{3}$			
$T_{3\text{ij}}(p)$	1	0	$0.584\,814\,781\,649\,163 \times 10^{3}$	4	3	$-0.587\,071\,076\,864\,459 \times 10^{-2}$
	2	1	$-0.616\,179\,320\,924\,617$	5	4	$0.515\,308\,185\,433\,082 \times 10^{-4}$
	3	2	$0.260\,763\,050\,899\,562$			
$T_{3\text{jk}}(p)$	1	0	$0.617\,229\,772\,068\,439 \times 10^{3}$	4	3	$-0.157\,391\,839\,848\,015 \times 10^{-1}$
	2	1	$-0.770\,600\,270\,141\,675 \times 10^{1}$	5	4	$0.137\,897\,492\,684\,194 \times 10^{-3}$
	3	2	$0.697\,072\,596\,851\,896$			
$T_{3\text{mn}}(p)$	1	0	$0.535\,339\,483\,742\,384 \times 10^{3}$	3	2	$-0.158\,365\,725\,441\,648$
	2	1	$0.761\,978\,122\,720\,128 \times 10^{1}$	4	3	$0.192\,871\,054\,508\,108 \times 10^{-2}$
$T_{3\text{op}}(p)$	1	0	$0.969\,461\,372\,400\,213 \times 10^{3}$	4	−1	$0.773\,845\,935\,768\,222 \times 10^{3}$
	2	1	$-0.332\,500\,170\,441\,278 \times 10^{3}$	5	−2	$-0.152\,313\,732\,937\,084 \times 10^{4}$
	3	2	$0.642\,859\,598\,466\,067 \times 10^{2}$			
$T_{3\text{qu}}(p)$	1	0	$0.565\,603\,648\,239\,126 \times 10^{3}$	3	2	$-0.102\,020\,639\,611\,016$
	2	1	$0.529\,062\,258\,221\,222 \times 10^{1}$	4	3	$0.122\,240\,301\,070\,145 \times 10^{-2}$
$T_{3\text{rx}}(p)$	1	0	$0.584\,561\,202\,520\,006 \times 10^{3}$	3	2	$0.243\,293\,362\,700\,452$
	2	1	$-0.102\,961\,025\,163\,669 \times 10^{1}$	4	3	$-0.294\,905\,044\,740\,799 \times 10^{-2}$

The following description of the use of the subregion boundary equations is summarized in Table 2 and Figures 3 and 4.

Table 2. Pressure ranges and corresponding subregion boundary equations for determining the correct subregion, 3a to 3t, for the backward equations $v(p,T)$

Pressure Range	Sub-region	For	Sub-region	For
$40\ \text{MPa} < p \le 100\ \text{MPa}$	3a	$T \le T_{3ab}(p)$	3b	$T > T_{3ab}(p)$
$25\ \text{MPa} < p \le 40\ \text{MPa}$	3c	$T \le T_{3cd}(p)$	3e	$T_{3ab}(p) < T \le T_{3ef}(p)$
	3d	$T_{3cd}(p) < T \le T_{3ab}(p)$	3f	$T > T_{3ef}(p)$
$23.5\ \text{MPa} < p \le 25\ \text{MPa}$	3c	$T \le T_{3cd}(p)$	3i	$T_{3ef}(p) < T \le T_{3ij}(p)$
	3g	$T_{3cd}(p) < T \le T_{3gh}(p)$	3j	$T_{3ij}(p) < T \le T_{3jk}(p)$
	3h	$T_{3gh}(p) < T \le T_{3ef}(p)$	3k	$T > T_{3jk}(p)$
$23\ \text{MPa} < p \le 23.5\ \text{MPa}$	3c	$T \le T_{3cd}(p)$	3i	$T_{3ef}(p) < T \le T_{3ij}(p)$
	3l	$T_{3cd}(p) < T \le T_{3gh}(p)$	3j	$T_{3ij}(p) < T \le T_{3jk}(p)$
	3h	$T_{3gh}(p) < T \le T_{3ef}(p)$	3k	$T > T_{3jk}(p)$
$22.5\ \text{MPa} < p \le 23\ \text{MPa}$	3c	$T \le T_{3cd}(p)$	3o	$T_{3ef}(p) < T \le T_{3op}(p)$
	3l	$T_{3cd}(p) < T \le T_{3gh}(p)$	3p	$T_{3op}(p) < T \le T_{3ij}(p)$
	3m	$T_{3gh}(p) < T \le T_{3mn}(p)$	3j	$T_{3ij}(p) < T \le T_{3jk}(p)$
	3n	$T_{3mn}(p) < T \le T_{3ef}(p)$	3k	$T > T_{3jk}(p)$
$p_{sat}^{97}(643.15\ \text{K}) < p \le 22.5\ \text{MPa}$	3c	$T \le T_{3cd}(p)$	3r	$T_{3rx}(p) < T \le T_{3ik}(p)$
	3q	$T_{3cd}(p) < T \le T_{3qu}(p)$	3k	$T > T_{3jk}(p)$
$20.5\ \text{MPa} < p \le p_{sat}^{97}(643.15\ \text{K})$	3c	$T \le T_{3cd}(p)$	3r	$T_{sat}^{97}(p) \le T \le T_{3jk}(p)$
	3s	$T_{3cd}(p) < T \le T_{sat}^{97}(p)$	3k	$T > T_{3jk}(p)$
p_{3cd}[b] $< p \le 20.5\ \text{MPa}$	3c	$T \le T_{3cd}(p)$	3t	$T \ge T_{sat}^{97}(p)$
	3s	$T_{3cd}(p) < T \le T_{sat}^{97}(p)$		
$p_{sat}^{97}(623.15\ \text{K}) < p \le p_{3cd}$[b]	3c	$T \le T_{sat}^{97}(p)$	3t	$T \ge T_{sat}^{97}(p)$

[b] $p_{3cd} = 1.900\ 881\ 189\ 173\ 929 \times 10^{1}$ MPa

The equation $T_{3ab}(p)$ approximates the critical isentrope from 25 MPa to 100 MPa. If the given temperature T is greater than $T_{3ab}(p)$ calculated from the given pressure p, then the point to be calculated is located in subregions 3b or 3e, otherwise it is in subregions 3a or 3d.

The equation $T_{3cd}(p)$ ranges from $p_{3cd} = 1.900\ 881\ 189\ 173\ 929 \times 10^{1}$ MPa to 40 MPa. The pressure of $p_{3cd} = 1.900\ 881\ 189\ 173\ 929 \times 10^{1}$ MPa is given as $T_{sat}^{97}(p) - T_{3cd}(p) = 0$. If the given temperature T is greater than $T_{3cd}(p)$ calculated from the given pressure p, then the point is located in subregions 3d, 3g, 3l, 3q or 3s, otherwise it is in subregion 3c.

The equation $T_{3gh}(p)$ ranges from 22.5 MPa to 25 MPa. If the given temperature T is greater than $T_{3gh}(p)$ calculated from the given pressure p, then the point is located in subregions 3h, or 3m, otherwise it is in subregions 3g or 3l.

The equation $T_{3ij}(p)$ approximates the isochore $v = 0.0041\ \text{m}^3\ \text{kg}^{-1}$ from 22.5 MPa to 25 MPa. If the given temperature T is greater than $T_{3ij}(p)$ calculated from the given pressure p, then the point is located in subregion 3j, otherwise it is in subregions 3i or 3p.

The equation $T_{3jk}(p)$ approximates the isochore $v = v''(20.5\,\mathrm{MPa})$ from 20.5 MPa to 25 MPa. If the given temperature T is greater than $T_{3jk}(p)$ calculated from the given pressure p, then the point is located in subregion 3k, otherwise it is in subregions 3j or 3r.

The equation $T_{3mn}(p)$ approximates the isochore $v = 0.0028\,\mathrm{m^3\,kg^{-1}}$ from 22.5 MPa to 23 MPa. If the given temperature T is greater than $T_{3mn}(p)$ calculated from the given pressure p, then the point is located in subregion 3n, otherwise it is in subregion 3m.

The equation $T_{3op}(p)$ approximates the isochore $v = 0.0034\,\mathrm{m^3\,kg^{-1}}$ from 22.5 MPa to 23 MPa. If the given temperature T is greater than $T_{3op}(p)$ calculated from the given pressure p, then the point is located in subregion 3p, otherwise it is in subregion 3o.

The equation $T_{3qu}(p)$ approximates the isochore $v = v'(643.15\,\mathrm{K})$ from $p = p_{\mathrm{sat}}^{97}(643.15\,\mathrm{K})$ where $p_{\mathrm{sat}}^{97}(643.15\,\mathrm{K}) = 2.104\,336\,732\times10^{1}\,\mathrm{MPa}$ to 22.5 MPa. If the given temperature T is less than or equal to $T_{3qu}(p)$ calculated from the given pressure p, then the point is located in subregion 3q.

The equation $T_{3rx}(p)$ approximates the isochore $v = v''(643.15\,\mathrm{K})$ from $p = p_{\mathrm{sat}}^{97}(643.15\,\mathrm{K})$ where $p_{\mathrm{sat}}^{97}(643.15\,\mathrm{K}) = 2.104\,336\,732\times10^{1}\,\mathrm{MPa}$ to 22.5 MPa. If the given temperature T is greater than $T_{3rx}(p)$ calculated from the given pressure p, then the point is located in subregion 3r.

The subregion boundary equation $T_{3ef}(p)$ is a straight line from 22.064 MPa to 40 MPa having the slope of the saturation-temperature curve of IAPWS-IF97 at the critical point. If the given temperature T is greater than $T_{3ef}(p)$ calculated from the given pressure p, then the point is located in subregions 3f, 3i or 3o, otherwise it is in subregions 3e, 3h or 3n.

Computer-program verification

To assist the user in computer-program verification of the equations for the subregion boundaries, Table 3 contains test values for calculated temperatures.

Table 3. Selected temperature values calculated from the subregion boundary equations [c]

Equation	p / MPa	T / K	Equation	p / MPa	T / K
$T_{3ab}(p)$	40	$6.930\,341\,408\times10^{2}$	$T_{3jk}(p)$	23	$6.558\,338\,344\times10^{2}$
$T_{3cd}(p)$	25	$6.493\,659\,208\times10^{2}$	$T_{3mn}(p)$	22.8	$6.496\,054\,133\times10^{2}$
$T_{3ef}(p)$	40	$7.139\,593\,992\times10^{2}$	$T_{3op}(p)$	22.8	$6.500\,106\,943\times10^{2}$
$T_{3gh}(p)$	23	$6.498\,873\,759\times10^{2}$	$T_{3qu}(p)$	22	$6.456\,355\,027\times10^{2}$
$T_{3ij}(p)$	23	$6.515\,778\,091\times10^{2}$	$T_{3rx}(p)$	22	$6.482\,622\,754\times10^{2}$

[c] It is recommended that programmed functions be verified using 8 byte real values for all variables.

5.2 Backward Equations $v(p,T)$ for the Subregions 3a to 3t

The backward equations $v(p,T)$ for the subregions 3a to 3t, except for 3n, have the following dimensionless form:

$$\frac{v(p,T)}{v^*} = \omega(\pi,\theta) = \left[\sum_{i=1}^{N} n_i \left[(\pi-a)^c\right]^{I_i} \left[(\theta-b)^d\right]^{J_i}\right]^e . \tag{4}$$

The equation for subregion 3n has the form:

$$\frac{v_{3n}(p,T)}{v^*} = \omega_{3n}(\pi,\theta) = \exp\left\{\sum_{i=1}^{N} n_i (\pi-a)^{I_i} (\theta-b)^{J_i}\right\}, \tag{5}$$

with $\omega = v/v^*$, $\pi = p/p^*$, and $\theta = T/T^*$. The reducing quantities v^*, p^*, and T^*, the number of coefficients N, the non-linear parameters a and b, and the exponents c, d, and e are listed in Table 4 for the equations of the subregions 3a to 3t. The coefficients n_i and exponents I_i and J_i of these equations are listed in Tables A1.1 to A1.20 of the Appendix.

Table 4. Reducing quantities v^*, p^*, and T^*, number of coefficients N, non-linear parameters a and b, and exponents c, d, and e for the $v(p,T)$ equations of the subregions 3a to 3t

Subregion	v^* $m^3\,kg^{-1}$	p^* MPa	T^* K	N	a	b	c	d	e
3a	0.0024	100	760	30	0.085	0.817	1	1	1
3b	0.0041	100	860	32	0.280	0.779	1	1	1
3c	0.0022	40	690	35	0.259	0.903	1	1	1
3d	0.0029	40	690	38	0.559	0.939	1	1	4
3e	0.0032	40	710	29	0.587	0.918	1	1	1
3f	0.0064	40	730	42	0.587	0.891	0.5	1	4
3g	0.0027	25	660	38	0.872	0.971	1	1	4
3h	0.0032	25	660	29	0.898	0.983	1	1	4
3i	0.0041	25	660	42	0.910	0.984	0.5	1	4
3j	0.0054	25	670	29	0.875	0.964	0.5	1	4
3k	0.0077	25	680	34	0.802	0.935	1	1	1
3l	0.0026	24	650	43	0.908	0.989	1	1	4
3m	0.0028	23	650	40	1.00	0.997	1	0.25	1
3n	0.0031	23	650	39	0.976	0.997	-	-	-
3o	0.0034	23	650	24	0.974	0.996	0.5	1	1
3p	0.0041	23	650	27	0.972	0.997	0.5	1	1
3q	0.0022	23	650	24	0.848	0.983	1	1	4
3r	0.0054	23	650	27	0.874	0.982	1	1	1
3s	0.0022	21	640	29	0.886	0.990	1	1	4
3t	0.0088	20	650	33	0.803	1.02	1	1	1

Computer-program verification

To assist the user in computer-program verification of the equations for the subregions 3a to 3t, Table 5 contains test values for calculated specific volumes.

Table 5. Selected specific volume values calculated from the equations for the subregions 3a to 3t [d]

Equation	p MPa	T K	v $m^3\ kg^{-1}$	Equation	p MPa	T K	v $m^3\ kg^{-1}$
$v_{3a}(p,T)$	50	630	$1.470\ 853\ 100 \times 10^{-3}$	$v_{3k}(p,T)$	23	660	$6.109\ 525\ 997 \times 10^{-3}$
	80	670	$1.503\ 831\ 359 \times 10^{-3}$		24	670	$6.427\ 325\ 645 \times 10^{-3}$
$v_{3b}(p,T)$	50	710	$2.204\ 728\ 587 \times 10^{-3}$	$v_{3l}(p,T)$	22.6	646	$2.117\ 860\ 851 \times 10^{-3}$
	80	750	$1.973\ 692\ 940 \times 10^{-3}$		23	646	$2.062\ 374\ 674 \times 10^{-3}$
$v_{3c}(p,T)$	20	630	$1.761\ 696\ 406 \times 10^{-3}$	$v_{3m}(p,T)$	22.6	648.6	$2.533\ 063\ 780 \times 10^{-3}$
	30	650	$1.819\ 560\ 617 \times 10^{-3}$		22.8	649.3	$2.572\ 971\ 781 \times 10^{-3}$
$v_{3d}(p,T)$	26	656	$2.245\ 587\ 720 \times 10^{-3}$	$v_{3n}(p,T)$	22.6	649.0	$2.923\ 432\ 711 \times 10^{-3}$
	30	670	$2.506\ 897\ 702 \times 10^{-3}$		22.8	649.7	$2.913\ 311\ 494 \times 10^{-3}$
$v_{3e}(p,T)$	26	661	$2.970\ 225\ 962 \times 10^{-3}$	$v_{3o}(p,T)$	22.6	649.1	$3.131\ 208\ 996 \times 10^{-3}$
	30	675	$3.004\ 627\ 086 \times 10^{-3}$		22.8	649.9	$3.221\ 160\ 278 \times 10^{-3}$
$v_{3f}(p,T)$	26	671	$5.019\ 029\ 401 \times 10^{-3}$	$v_{3p}(p,T)$	22.6	649.4	$3.715\ 596\ 186 \times 10^{-3}$
	30	690	$4.656\ 470\ 142 \times 10^{-3}$		22.8	650.2	$3.664\ 754\ 790 \times 10^{-3}$
$v_{3g}(p,T)$	23.6	649	$2.163\ 198\ 378 \times 10^{-3}$	$v_{3q}(p,T)$	21.1	640	$1.970\ 999\ 272 \times 10^{-3}$
	24	650	$2.166\ 044\ 161 \times 10^{-3}$		21.8	643	$2.043\ 919\ 161 \times 10^{-3}$
$v_{3h}(p,T)$	23.6	652	$2.651\ 081\ 407 \times 10^{-3}$	$v_{3r}(p,T)$	21.1	644	$5.251\ 009\ 921 \times 10^{-3}$
	24	654	$2.967\ 802\ 335 \times 10^{-3}$		21.8	648	$5.256\ 844\ 741 \times 10^{-3}$
$v_{3i}(p,T)$	23.6	653	$3.273\ 916\ 816 \times 10^{-3}$	$v_{3s}(p,T)$	19.1	635	$1.932\ 829\ 079 \times 10^{-3}$
	24	655	$3.550\ 329\ 864 \times 10^{-3}$		20	638	$1.985\ 387\ 227 \times 10^{-3}$
$v_{3j}(p,T)$	23.5	655	$4.545\ 001\ 142 \times 10^{-3}$	$v_{3t}(p,T)$	17	626	$8.483\ 262\ 001 \times 10^{-3}$
	24	660	$5.100\ 267\ 704 \times 10^{-3}$		20	640	$6.227\ 528\ 101 \times 10^{-3}$

[d] It is recommended that programmed functions be verified using 8 byte real values for all variables.

5.3 Calculation of Thermodynamic Properties with the $v(p,T)$ Backward Equations

The $v(p,T)$ backward equations described in Section 5.2 together with IAPWS-IF97 basic equation $f(v,T)$ make it possible to determine all thermodynamic properties, *e.g.*, enthalpy, entropy, isobaric heat capacity, speed of sound, from pressure p and temperature T in region 3 without iteration.

The following steps should be made:

- Identify the subregion (3a to 3t) for given pressure p and temperature T following the instructions of Section 5.1 in conjunction with Table 2 and Figures 3 and 4. Then, calculate the specific volume v for the subregion using the corresponding backward equation $v(p,T)$.
- Calculate the desired thermodynamic property from the previously calculated specific volume v and the given temperature T using the derivatives of the IAPWS-IF97 basic equation $f(v,T)$, where $v = v(p,T)$; see Table 31 in [1].

5.4 Numerical Consistency

5.4.1 Numerical Consistency with the Basic Equation of IAPWS-IF97

The maximum relative deviations and root-mean-square relative deviations of specific volume, calculated from the backward equations $v(p,T)$ for subregions 3a to 3t, from the IAPWS-IF97 basic equation $f(v,T)$ in comparison with the permissible tolerances are listed in Table 6. The calculation of the root-mean-square values is described in Section 1.

Table 6 also contains the maximum relative deviations and root-mean-square relative deviations of specific enthalpy, specific entropy, specific isobaric heat capacity, and speed of sound, calculated as described in Section 5.3.

Table 6. Maximum relative deviations and root-mean-square relative deviations of the specific volume, calculated from the backward equations for subregions 3a to 3t, and maximum relative deviations of specific enthalpy, specific entropy, specific isobaric heat capacity and speed of sound, calculated as described in Section 5.3, from the IAPWS-IF97 basic equation $f(v,T)$

Subregion	$\lvert\Delta v/v\rvert$ %		$\lvert\Delta h/h\rvert$ %		$\lvert\Delta s/s\rvert$ %		$\lvert\Delta c_p/c_p\rvert$ %		$\lvert\Delta w/w\rvert$ %	
	max	RMS	max	RMS	max	RMS	max	RMS	max	RMS
3a	0.00061	0.00031	0.00018	0.00008	0.00026	0.00011	0.0016	0.0006	0.0015	0.0006
3b	0.00064	0.00035	0.00017	0.00008	0.00016	0.00008	0.0012	0.0003	0.0008	0.0003
3c	0.00080	0.00038	0.00026	0.00012	0.00025	0.00011	0.0059	0.0016	0.0023	0.0010
3d	0.00059	0.00025	0.00018	0.00008	0.00014	0.00006	0.0035	0.0010	0.0012	0.0004
3e	0.00072	0.00033	0.00018	0.00009	0.00014	0.00007	0.0017	0.0005	0.0006	0.0002
3f	0.00068	0.00020	0.00018	0.00005	0.00013	0.00004	0.0015	0.0003	0.0002	0.0001
3g	0.00047	0.00016	0.00014	0.00005	0.00011	0.00004	0.0032	0.0011	0.0010	0.0003
3h	0.00085	0.00044	0.00022	0.00012	0.00017	0.00009	0.0066	0.0018	0.0006	0.0002
3i	0.00067	0.00028	0.00018	0.00008	0.00013	0.00006	0.0019	0.0006	0.0002	0.0001
3j	0.00034	0.00019	0.00009	0.00005	0.00007	0.00004	0.0020	0.0006	0.0002	0.0001
3k	0.00034	0.00012	0.00008	0.00003	0.00007	0.00002	0.0018	0.0003	0.0002	0.0001
3l	0.00033	0.00019	0.00010	0.00006	0.00008	0.00005	0.0035	0.0015	0.0008	0.0004
3m	0.00057	0.00031	0.00015	0.00009	0.00011	0.00006	0.0062	0.0030	0.0006	0.0002
3n	0.00064	0.00029	0.00017	0.00008	0.00012	0.00006	0.0050	0.0013	0.0002	0.0001
3o	0.00031	0.00015	0.00008	0.00004	0.00006	0.00003	0.0007	0.0002	0.0001	0.0001
3p	0.00044	0.00022	0.00012	0.00006	0.00009	0.00005	0.0026	0.0010	0.0002	0.0001
3q	0.00036	0.00018	0.00012	0.00006	0.00009	0.00005	0.0040	0.0016	0.0010	0.0005
3r	0.00037	0.00007	0.00010	0.00002	0.00008	0.00002	0.0030	0.0004	0.0002	0.0001
3s	0.00030	0.00016	0.00010	0.00005	0.00007	0.00004	0.0033	0.0015	0.0009	0.0005
3t	0.00095	0.00045	0.00022	0.00010	0.00018	0.00008	0.0046	0.0015	0.0004	0.0002
permissible tolerance	0.001		0.001		0.001		0.01		0.01	

Table 6 shows that the deviations of the specific volume, specific enthalpy, and specific entropy from the IAPWS-IF97 basic equation are less than 0.001 % and the deviations of specific isobaric heat capacity and speed of sound are less than 0.01 %. Therefore, the values of specific volume, specific enthalpy and specific entropy of IAPWS-IF97 are represented with 5 significant figures, and the values of specific isobaric heat capacity and speed of sound with 4 significant figures by using the backward equations $v(p,T)$.

5.4.2 Consistency at Boundaries Between Subregions

The maximum relative differences of specific volume between the $v(p,T)$ backward equations of adjacent subregions along the subregion boundary pressures are listed in the third column of Table 7. Table 8 contains these maximum relative differences along the subregion boundary equations.

Table 7. Maximum relative deviations of specific volume between the backward equations $v(p,T)$ of adjacent subregions and maximum relative deviations of specific enthalpy, specific entropy, specific isobaric heat capacity, and speed of sound, calculated as described in Section 5.3, along the subregion boundary pressures

Subregion Boundary	Between Subregions	$\lvert\Delta v/v\rvert_{max}$ %	$\lvert\Delta h/h\rvert_{max}$ %	$\lvert\Delta s/s\rvert_{max}$ %	$\lvert\Delta c_p/c_p\rvert_{max}$ %	$\lvert\Delta w/w\rvert_{max}$ %
p = 40 MPa	3a, 3c	0.00074	0.00021	0.00028	0.0018	0.0019
	3a, 3d	0.00060	0.00017	0.00013	0.0013	0.0006
	3b, 3e	0.00062	0.00015	0.00012	0.0009	0.0004
	3b, 3f	0.00078	0.00018	0.00014	0.0004	0.0002
p = 25 MPa	3d, 3g	0.00056	0.00015	0.00011	0.0031	0.0010
	3d, 3h	0.00056	0.00015	0.00011	0.0021	0.0003
	3e, 3h	0.00063	0.00017	0.00013	0.0014	0.0002
	3f, 3i	0.00055	0.00014	0.00011	0.0011	0.0002
	3f, 3j	0.00060	0.00015	0.00011	0.0015	0.0002
	3f, 3k	0.00064	0.00013	0.00011	0.0011	0.0002
p = 23.5 MPa	3g, 3l	0.00049	0.00015	0.00012	0.0033	0.0011
p = 23 MPa	3h, 3m	0.00084	0.00023	0.00017	0.0074	0.0007
	3h, 3n	0.00085	0.00022	0.00016	0.0047	0.0003
	3i, 3o	0.00047	0.00012	0.00009	0.0006	0.0002
	3i, 3p	0.00059	0.00015	0.00012	0.0020	0.0002
p = 22.5 MPa	3l, 3q	0.00033	0.00010	0.00008	0.0025	0.0008
	3j, 3r	0.00035	0.00009	0.00007	0.0015	0.0002
$p = p_{sat}^{97}(643.15\ K)$	3q, 3s	0.00033	0.00010	0.00008	0.0036	0.0008
p = 20.5 MPa	3k, 3t	0.00042	0.00009	0.00008	0.0019	0.0002
permissible tolerance		0.001	0.001	0.001	0.01	0.01

Table 8. Maximum relative deviations of specific volume between the backward equations $v(p,T)$ of the adjacent subregions and maximum relative deviations of specific enthalpy, specific entropy, specific isobaric heat capacity, and speed of sound, calculated as described in Section 5.3, along the subregion boundary equations

Subregion Boundary Equation	Between Subregions	$\lvert\Delta v/v\rvert_{max}$ %	$\lvert\Delta h/h\rvert_{max}$ %	$\lvert\Delta s/s\rvert_{max}$ %	$\lvert\Delta c_p/c_p\rvert_{max}$ %	$\lvert\Delta w/w\rvert_{max}$ %
$T_{3ab}(p)$	3a, 3b	0.00075	0.00020	0.00020	0.0012	0.0010
	3d, 3e	0.00061	0.00017	0.00013	0.0016	0.0005
$T_{3cd}(p)$	3c, 3d	0.00089	0.00027	0.00021	0.0040	0.0016
	3c, 3g	0.00029	0.00009	0.00007	0.0017	0.0007
	3c, 3l	0.00059	0.00019	0.00014	0.0039	0.0015
	3c, 3q	0.00056	0.00018	0.00014	0.0040	0.0015
	3c, 3s	0.00039	0.00012	0.00010	0.0031	0.0011
$T_{3ef}(p)$	3e, 3f	0.00060	0.00016	0.00012	0.0005	0.0001
	3h, 3i	0.00061	0.00016	0.00012	0.0007	0.0001
	3n, 3o	0.00031	0.00008	0.00006	0.0004	0.0001
$T_{3gh}(p)$	3g, 3h	0.00083	0.00022	0.00016	0.0058	0.0006
	3l, 3h	0.00083	0.00022	0.00016	0.0064	0.0006
	3l, 3m	0.00052	0.00014	0.00011	0.0058	0.0006
$T_{3ij}(p)$	3i, 3j	0.00034	0.00009	0.00007	0.0010	0.0002
	3p, 3j	0.00036	0.00009	0.00007	0.0020	0.0002
$T_{3jk}(p)$	3j, 3k	0.00030	0.00007	0.00006	0.0008	0.0001
	3r, 3k	0.00029	0.00007	0.00006	0.0018	0.0002
$T_{3mn}(p)$	3m, 3n	0.00090	0.00024	0.00017	0.0070	0.0003
$T_{3op}(p)$	3o, 3p	0.00041	0.00011	0.00008	0.0013	0.0002
permissible tolerance		0.001	0.001	0.001	0.01	0.01

For example, the maximum relative difference between the backward equation of subregion 3a and the backward equation of subregion 3b along the subregion boundary $T_{3ab}(p)$ was determined as follows:

$$\left|\frac{\Delta v}{v}\right|_{max} = \left|\frac{v_{3a}\left(p,T_{3ab}(p)\right) - v_{3b}\left(p,T_{3ab}(p)\right)}{v_{3b}\left(p,T_{3ab}(p)\right)}\right|_{max} .$$

In addition, Tables 7 and 8 contain the maximum relative differences of specific enthalpy, specific entropy, specific isobaric heat capacity and speed of sound, calculated as described in Section 5.3, along the subregion boundaries of the $v(p,T)$ backward equations. For example, the maximum relative difference of specific enthalpy along the subregion boundary $T_{3ab}(p)$ was determined as follows:

$$\left|\frac{\Delta h}{h}\right|_{max} = \left|\frac{h_3^{97}(v_{3a},T_{3ab}) - h_3^{97}(v_{3b},T_{3ab})}{h_3^{97}(v_{3b},T_{3ab})}\right|_{max}$$

where $v_{3a} = v_{3a}\left(p,T_{3ab}(p)\right)$ and $v_{3b} = v_{3b}\left(p,T_{3ab}(p)\right)$.

Tables 7 and 8 show that the relative specific volume differences between the backward equations $v(p,T)$ of the adjacent subregions and the maximum relative deviations of specific enthalpy, specific entropy, specific isobaric heat capacity, and speed of sound along the subregion boundary pressures and along the subregion boundary equations are smaller than the permissible numerical tolerances of these equations with the IAPWS-IF97 basic equation.

6 Auxiliary Equations $v(p,T)$ for the Region very close to the Critical Point

6.1 Subregions

The auxiliary equations $v(p,T)$ for the subregions 3u to 3z are valid from

$$T_{3\mathrm{qu}}(p) < T \le T_{3\mathrm{rx}}(p) \text{ for } p_{\mathrm{sat}}^{97}(643.15\ \mathrm{K}) < p \le 22.5\ \mathrm{MPa}\text{ ; see Figure 5.}$$

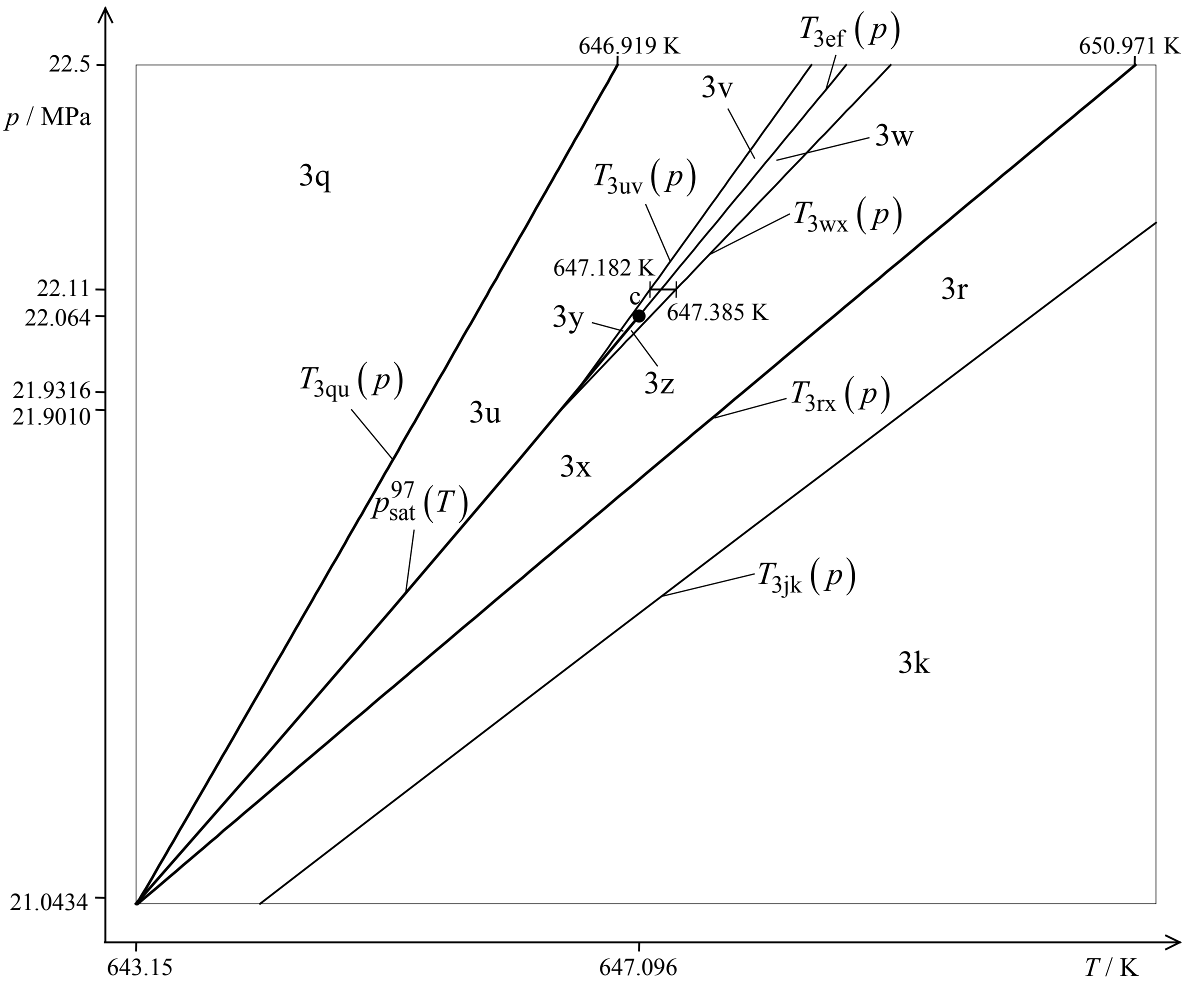

Figure 5. Division of region 3 into subregions 3u to 3z for the auxiliary equations

The subregion boundary equation $T_{3\text{uv}}(p)$ has the form of Eq. (1) and $T_{3\text{wx}}(p)$ has the form of Eq. (2). The coefficients n_i and the exponents I_i of the boundary equations are listed in Table 9.

Table 9. Numerical values of the coefficients of the equations $T_{3\text{uv}}(p)$ and $T_{3\text{wx}}(p)$ for subregion boundaries

Equation	i	I_i	n_i	i	I_i	n_i
$T_{3\text{uv}}(p)$	1	0	0.528 199 646 263 062 × 10^3	3	2	−0.222 814 134 903 755
	2	1	0.890 579 602 135 307 × 10^1	4	3	0.286 791 682 263 697 × 10^−2
$T_{3\text{wx}}(p)$	1	0	0.728 052 609 145 380 × 10^1	4	−1	0.329 196 213 998 375 × 10^3
	2	1	0.973 505 869 861 952 × 10^2	5	−2	0.873 371 668 682 417 × 10^3
	3	2	0.147 370 491 183 191 × 10^2			

The following description of the use of the subregion boundary equations is summarized in Table 10 and Figure 5.

Table 10. Pressure ranges and corresponding subregion boundary equations for determining the correct subregion, 3u to 3z, for the auxiliary equations $v(p,T)$

Supercritical Pressure Region				
Pressure Range	Sub-region	For	Sub-region	For
$22.11\ \text{MPa} < p \le 22.5\ \text{MPa}$	3u	$T_{3\text{qu}}(p) < T \le T_{3\text{uv}}(p)$	3v	$T_{3\text{uv}}(p) < T \le T_{3\text{ef}}(p)$
	3w	$T_{3\text{ef}}(p) < T \le T_{3\text{wx}}(p)$	3x	$T_{3\text{wx}}(p) < T \le T_{3\text{rx}}(p)$
$22.064\ \text{MPa} < p \le 22.11\ \text{MPa}$	3u	$T_{3\text{qu}}(p) < T \le T_{3\text{uv}}(p)$	3y	$T_{3\text{uv}}(p) < T \le T_{3\text{ef}}(p)$
	3z	$T_{3\text{ef}}(p) < T \le T_{3\text{wx}}(p)$	3x	$T_{3\text{wx}}(p) < T \le T_{3\text{rx}}(p)$

Subcritical Pressure Region			
Temperature Range	Pressure Range	Sub-region	For
$T \le T_{\text{sat}}^{97}(p)$	$p_{\text{sat}}^{97}\left(0.00264\ \text{m}^3\ \text{kg}^{-1}\right)$ [e] $< p \le 22.064\,\text{MPa}$	3u	$T_{3\text{qu}}(p) < T \le T_{3\text{uv}}(p)$
		3y	$T_{3\text{uv}}(p) < T$
	$p_{\text{sat}}^{97}(643.15\ \text{K}) < p \le p_{\text{sat}}^{97}\left(0.00264\ \text{m}^3\ \text{kg}^{-1}\right)$ [e]	3u	$T_{3\text{qu}}(p) < T$
$T \ge T_{\text{sat}}^{97}(p)$	$p_{\text{sat}}^{97}\left(0.00385\ \text{m}^3\ \text{kg}^{-1}\right)$ [f] $< p \le 22.064\,\text{MPa}$	3z	$T \le T_{3\text{wx}}(p)$
		3x	$T_{3\text{wx}}(p) < T \le T_{3\text{rx}}(p)$
	$p_{\text{sat}}^{97}(643.15\ \text{K}) < p \le p_{\text{sat}}^{97}\left(0.00385\ \text{m}^3\ \text{kg}^{-1}\right)$ [f]	3x	$T \le T_{3\text{rx}}(p)$

[e] $p_{\text{sat}}^{97}\left(0.00264\ \text{m}^3\ \text{kg}^{-1}\right) = 2.193\ 161\ 551 \times 10^1\ \text{MPa}$

[f] $p_{\text{sat}}^{97}\left(0.00385\ \text{m}^3\ \text{kg}^{-1}\right) = 2.190\ 096\ 265 \times 10^1\ \text{MPa}$

The equation $T_{3\mathrm{uv}}(p)$ approximates the isochore $v = 0.00264\ \mathrm{m^3\ kg^{-1}}$ from $p = p_{\mathrm{sat}}^{97}\left(0.00264\ \mathrm{m^3\ kg^{-1}}\right)$ where $p_{\mathrm{sat}}^{97}\left(0.00264\ \mathrm{m^3\ kg^{-1}}\right) = 2.193\,161\,551\times10^{1}\ \mathrm{MPa}$ to 22.5 MPa. If the given temperature T is greater than $T_{3\mathrm{uv}}(p)$ calculated from the given pressure p, then the point is located in subregions 3v or 3y, otherwise it is in subregion 3u.

The equation $T_{3\mathrm{wx}}(p)$ approximates the isochore $v = 0.00385\ \mathrm{m^3\ kg^{-1}}$ from $p = p_{\mathrm{sat}}^{97}\left(0.00385\ \mathrm{m^3\ kg^{-1}}\right)$ where $p_{\mathrm{sat}}^{97}\left(0.00385\ \mathrm{m^3\ kg^{-1}}\right) = 2.190\,096\,265\times10^{1}\ \mathrm{MPa}$ to 22.5 MPa. If the given temperature T is greater than $T_{3\mathrm{wx}}(p)$ calculated from the given pressure p, then the point is located in subregion 3x, otherwise it is in subregions 3w or 3z.

Computer-program verification

To assist the user in computer-program verification of the equations for the subregion boundaries, Table 11 contains test values for calculated temperatures.

Table 11. Selected temperature values calculated from the subregion boundary equations $T_{3\mathrm{uv}}(p)$ and $T_{3\mathrm{wx}}(p)$ [g]

Equation	p MPa	T K
$T_{3\mathrm{uv}}(p)$	22.3	$6.477\,996\,121 \times 10^{2}$
$T_{3\mathrm{wx}}(p)$	22.3	$6.482\,049\,480 \times 10^{2}$

[g] It is recommended that programmed functions be verified using 8 byte real values for all variables.

6.2 Auxiliary Equations $v(p,T)$ for the Subregions 3u to 3z

The auxiliary equations $v(p,T)$ for the subregions 3u to 3z have the dimensionless form of Eq. (4). The reducing quantities v^*, p^*, and T^*, the number of coefficients *N*, the non-linear parameters *a* and *b*, and the exponents *c*, *d*, and *e* are listed in Table 12 for the auxiliary equations of the subregions 3u to 3z. The coefficients n_i and exponents I_i and J_i are listed in Tables A2.1 to A2.6 of the Appendix.

Table 12. Reducing quantities v^*, p^*, and T^*, number of coefficients *N*, non-linear parameters *a* and *b*, and exponents *c*, *d*, and *e* for the auxiliary equations $v(p,T)$ of the subregions 3u to 3z

Subregion	v^* $\mathrm{m^3\ kg^{-1}}$	p^* MPa	T^* K	N	a	b	c	d	e
3u	0.0026	23	650	38	0.902	0.988	1	1	1
3v	0.0031	23	650	39	0.960	0.995	1	1	1
3w	0.0039	23	650	35	0.959	0.995	1	1	4
3x	0.0049	23	650	36	0.910	0.988	1	1	1
3y	0.0031	22	650	20	0.996	0.994	1	1	4
3z	0.0038	22	650	23	0.993	0.994	1	1	4

Computer-program verification

To assist the user in computer-program verification of the auxiliary equations for the subregions 3u to 3z, Table 13 contains test values for calculated specific volumes.

Table 13. Selected specific volume values calculated from the auxiliary equations for the subregions 3u to 3z [h]

Equation	p MPa	T K	v $m^3\ kg^{-1}$	Equation	p MPa	T K	v $m^3\ kg^{-1}$
$v_{3u}(p,T)$	21.5	644.6	$2.268\,366\,647 \times 10^{-3}$	$v_{3x}(p,T)$	22.11	648.0	$4.528\,072\,649 \times 10^{-3}$
	22.0	646.1	$2.296\,350\,553 \times 10^{-3}$		22.3	649.0	$4.556\,905\,799 \times 10^{-3}$
$v_{3v}(p,T)$	22.5	648.6	$2.832\,373\,260 \times 10^{-3}$	$v_{3y}(p,T)$	22.0	646.84	$2.698\,354\,719 \times 10^{-3}$
	22.3	647.9	$2.811\,424\,405 \times 10^{-3}$		22.064	647.05	$2.717\,655\,648 \times 10^{-3}$
$v_{3w}(p,T)$	22.15	647.5	$3.694\,032\,281 \times 10^{-3}$	$v_{3z}(p,T)$	22.0	646.89	$3.798\,732\,962 \times 10^{-3}$
	22.3	648.1	$3.622\,226\,305 \times 10^{-3}$		22.064	647.15	$3.701\,940\,010 \times 10^{-3}$

[h] It is recommended that programmed functions be verified using 8 byte real values for all variables.

6.3 Numerical Consistency

6.3.1 Numerical Consistency with the Basic Equation of IAPWS-IF97

The maximum relative differences and root-mean-square relative deviations of specific volume, calculated from the auxiliary equations $v(p,T)$ for subregions 3u to 3z, to the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ are listed in Table 14. For the calculation of the root-mean-square values, which is described in Section 1, one million points uniformly distributed over the range of validity in the p-T plane have been used.

Table 14 shows that the deviations of the specific volume from the IAPWS-IF97 basic equation are better than 0.1 %. Only in a small region for pressures less than 22.11 MPa (see Figure 5) do the deviations of the specific volume from the IAPWS-IF97 basic equation approach 2 %.

Table 14. Maximum relative deviations and root-mean-square relative deviations of the specific volume, calculated from the auxiliary equations for subregions 3u to 3z from the IAPWS-IF97 basic equation

Subregion	$\|\Delta v/v\|$ % max	$\|\Delta v/v\|$ % RMS	Subregion	$\|\Delta v/v\|$ % max	$\|\Delta v/v\|$ % RMS
3u	0.097	0.058	3x	0.090	0.050
3v	0.082	0.040	3y	1.77	1.04
3w	0.065	0.023	3z	1.80	0.921

6.3.2 Consistency at Boundaries Between Subregions

The maximum relative differences of specific volume between the $v(p,T)$ auxiliary equations of adjacent subregions along the subregion boundary pressures are listed in Table 15. Table 16 contains these maximum relative differences along the subregion boundary equations.

Table 15. Maximum relative deviations of specific volume between the auxiliary equations $v(p,T)$ of the adjacent subregions along the subregion boundary pressures

Subregion Boundary	Between Subregions	$\lvert \Delta v/v \rvert_{max}$ %
p = 22.5 MPa	3l, 3u	0.096
	3m, 3u	0.096
	3m, 3v	0.035
	3n, 3v	0.046
	3o, 3w	0.019
	3p, 3w	0.021
	3p, 3x	0.042
	3j, 3x	0.043
p = 22.11 MPa	3v, 3y	1.7
	3w, 3z	1.7

Table 16. Maximum relative deviations of specific volume between the auxiliary equations $v(p,T)$ of the adjacent subregions along the subregion boundary equations

Subregion Boundary Equation	Between Subregions	$\lvert \Delta v/v \rvert_{max}$ %
$T_{3qu}(p)$	3q, 3u	0.097
$T_{3rx}(p)$	3x, 3r	0.045
$T_{3uv}(p)$	3u, 3v	0.14
	3u, 3y	1.8
$T_{3ef}(p)$	3v, 3w	0.080
	3y, 3z	3.5
$T_{3wx}(p)$	3w, 3x	0.049
	3z, 3x	1.8

7 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the backward equations $v(p,T)$ was reducing the computing time to obtain thermodynamic properties and differential quotients from given variables (p,T) in region 3. Using IAPWS-IF97, time-consuming iteration is required. Using the $v(p,T)$ backward equations, iteration can be avoided. The calculation speed is about 17 times faster than iteration with IAPWS-IF97.

If iteration is used, the time to reach convergence can be significantly reduced by using the backward equations $v(p,T)$ to calculate very accurate starting values.

8 Application of the Backward and Auxiliary Equations $v(p,T)$

The numerical consistency of the specific volume v calculated from the main backward equations $v_3(p,T)$ described in Section 5 with the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ is sufficient for most applications in process modeling. For many calculations, the numerical consistency of the auxiliary equations described in Section 6 is also satisfactory in the region very close to the critical point.

For applications where the demands on numerical consistency are extremely high, iteration using the IAPWS-IF97 basic equation $f(v,T)$ may be necessary. In these cases, the backward and auxiliary equations $v(p,T)$ can be used for calculating very accurate starting values.

The backward and auxiliary equations $v(p,T)$ should only be used in their ranges of validity described in Section 4. They should not be used for determining any thermodynamic derivatives. They should also not be used together with the fundamental equation in iterative calculations of other backward functions such as $T(p,h)$ or $T(p,s)$. Iteration of backward functions can only be performed by using the fundamental equations.

In any case, depending on the application, a conscious decision is required whether to use the backward and auxiliary equations $v(p,T)$ or to calculate the corresponding values by iteration from the basic equation of IAPWS-IF97.

9 References

[1] International Association for the Properties of Water and Steam, *Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, IAPWS Secretariat, Electric Power Research Institute, Palo Alto, CA, 1997.

[2] Wagner, W., Cooper, J. R., Dittmann, A., Kijima, J., Kretzschmar, H.-J., Kruse, A., Mareš, R., Oguchi, K., Sato, H., Stöcker, I., Šifner, O., Tanishita, I., Trübenbach, J., and Willkommen, Th., The IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam, *Journal of Engineering for Gas Turbines and Power*, Vol. 122, 2000, pp. 150-182.

[3] International Association for the Properties of Water and Steam, *Supplementary Release on Backward Equations for Pressure as a Function of Enthalpy and Entropy p(h,s) to the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, IAPWS Secretariat, Electric Power Research Institute, Palo Alto, CA, 2001.

[4] Kretzschmar, H.-J., Cooper, J. R., Dittmann, A., Friend, D. G., Gallagher, J., Knobloch, K., Mareš, R., Miyagawa, K., Stöcker, I., Trübenbach, J., Wagner, W., and Willkommen, Th., Supplementary Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the Industrial Formulation IAPWS-IF97 for Water and Steam, *Journal of Engineering for Gas Turbines and Power,* in press.

[5] International Association for the Properties of Water and Steam, *Revised Supplementary Release on Backward Equations for the Functions T(p,h), v(p,h) and T(p,s), v(p,s) for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam,* IAPWS Secretariat, Electric Power Research Institute, Palo Alto, CA, 2004.

[6] Kretzschmar, H.-J., Cooper, J. R., Dittmann, A., Friend, D. G., Gallagher, J. S., Harvey, A. H., Knobloch, K., Mareš, R., Miyagawa, K., Okita, N., Stöcker, I., Wagner, W., and Weber, I., Supplementary Backward Equations $T(p,h)$, $v(p,h)$, and $T(p,s)$, $v(p,s)$ for the Critical and Supercritical Regions (Region 3) of the Industrial Formulation IAPWS-IF97 for Water and Steam, *Journal of Engineering for Gas Turbines and Power,* submitted.

[7] International Association for the Properties of Water and Steam, *Supplementary Release on Backward Equations p(h,s) for Region 3, Equations as a Function of h and s for the Region Boundaries, and an Equation $T_{sat}(h,s)$ for Region 4 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam,* IAPWS Secretariat, Electric Power Research Institute, Palo Alto, CA, 2004.

[8] Kretzschmar, H.-J., Cooper, J. R., Dittmann, A., Friend, D. G., Gallagher, J. S., Harvey, A. H., Knobloch, K., Mareš, R., Miyagawa, K., Okita, N., Span, R., Stöcker, I., Wagner, W., and Weber, I., Supplementary Backward Equations $p(h,s)$ for the Critical and Supercritical Regions (Region 3), Equations for the Region Boundaries, and an Equation for the Two-Phase Region of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam, *Journal of Engineering for Gas Turbines and Power*, to be submitted.

[9] Kretzschmar, H.-J., Cooper, J. R., Dittmann, A., Friend, D. G., Gallagher, J. S., Harvey, A. H., Knobloch, K., Mareš, R., Miyagawa, K., Okita, N., Span, R., Stöcker, I., Wagner, W., and Weber, I., Supplementary Backward Equations for Specific Volume as Function of Pressure and Temperature $v(p,T)$ for the Critical and Supercritical Regions (Region 3) of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam, *Journal of Engineering for Gas Turbines and Power*, in preparation.

[10] International Association for the Properties of Water and Steam, *IAPWS Release on the Values of Temperature, Pressure and Density of Ordinary and Heavy Water Substances at their Respective Critical Points (Revised 1992)*, IAPWS Secretariat, Electric Power Research Institute, Palo Alto, CA, 1992.

Appendix

A1 Coefficients for Backward Equations

Table A1.1. Coefficients and exponents of the backward equation $v_{3a}(p,T)$ for subregion 3a

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	5	$0.110\,879\,558\,823\,853 \times 10^{-2}$	16	−3	1	$-0.122\,494\,831\,387\,441 \times 10^{-1}$
2	−12	10	$0.572\,616\,740\,810\,616 \times 10^{3}$	17	−3	3	$0.179\,357\,604\,019\,989 \times 10^{1}$
3	−12	12	$-0.767\,051\,948\,380\,852 \times 10^{5}$	18	−3	6	$0.442\,729\,521\,058\,314 \times 10^{2}$
4	−10	5	$-0.253\,321\,069\,529\,674 \times 10^{-1}$	19	−2	0	$-0.593\,223\,489\,018\,342 \times 10^{-2}$
5	−10	10	$0.628\,008\,049\,345\,689 \times 10^{4}$	20	−2	2	$0.453\,186\,261\,685\,774$
6	−10	12	$0.234\,105\,654\,131\,876 \times 10^{6}$	21	−2	3	$0.135\,825\,703\,129\,140 \times 10^{1}$
7	−8	5	$0.216\,867\,826\,045\,856$	22	−1	0	$0.408\,748\,415\,856\,745 \times 10^{-1}$
8	−8	8	$-0.156\,237\,904\,341\,963 \times 10^{3}$	23	−1	1	$0.474\,686\,397\,863\,312$
9	−8	10	$-0.269\,893\,956\,176\,613 \times 10^{5}$	24	−1	2	$0.118\,646\,814\,997\,915 \times 10^{1}$
10	−6	1	$-0.180\,407\,100\,085\,505 \times 10^{-3}$	25	0	0	$0.546\,987\,265\,727\,549$
11	−5	1	$0.116\,732\,227\,668\,261 \times 10^{-2}$	26	0	1	$0.195\,266\,770\,452\,643$
12	−5	5	$0.266\,987\,040\,856\,040 \times 10^{2}$	27	1	0	$-0.502\,268\,790\,869\,663 \times 10^{-1}$
13	−5	10	$0.282\,776\,617\,243\,286 \times 10^{5}$	28	1	2	$-0.369\,645\,308\,193\,377$
14	−4	8	$-0.242\,431\,520\,029\,523 \times 10^{4}$	29	2	0	$0.633\,828\,037\,528\,420 \times 10^{-2}$
15	−3	0	$0.435\,217\,323\,022\,733 \times 10^{-3}$	30	2	2	$0.797\,441\,793\,901\,017 \times 10^{-1}$

Table A1.2. Coefficients and exponents of the backward equation $v_{3b}(p,T)$ for subregion 3b

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	10	$-0.827\,670\,470\,003\,621 \times 10^{-1}$	17	−3	2	$-0.416\,375\,290\,166\,236 \times 10^{-1}$
2	−12	12	$0.416\,887\,126\,010\,565 \times 10^{2}$	18	−3	3	$-0.413\,754\,957\,011\,042 \times 10^{2}$
3	−10	8	$0.483\,651\,982\,197\,059 \times 10^{-1}$	19	−3	5	$-0.506\,673\,295\,721\,637 \times 10^{2}$
4	−10	14	$-0.291\,032\,084\,950\,276 \times 10^{5}$	20	−2	0	$-0.572\,212\,965\,569\,023 \times 10^{-3}$
5	−8	8	$-0.111\,422\,582\,236\,948 \times 10^{3}$	21	−2	2	$0.608\,817\,368\,401\,785 \times 10^{1}$
6	−6	5	$-0.202\,300\,083\,904\,014 \times 10^{-1}$	22	−2	5	$0.239\,600\,660\,256\,161 \times 10^{2}$
7	−6	6	$0.294\,002\,509\,338\,515 \times 10^{3}$	23	−1	0	$0.122\,261\,479\,925\,384 \times 10^{-1}$
8	−6	8	$0.140\,244\,997\,609\,658 \times 10^{3}$	24	−1	2	$0.216\,356\,057\,692\,938 \times 10^{1}$
9	−5	5	$-0.344\,384\,158\,811\,459 \times 10^{3}$	25	0	0	$0.398\,198\,903\,368\,642$
10	−5	8	$0.361\,182\,452\,612\,149 \times 10^{3}$	26	0	1	$-0.116\,892\,827\,834\,085$
11	−5	10	$-0.140\,699\,677\,420\,738 \times 10^{4}$	27	1	0	$-0.102\,845\,919\,373\,532$
12	−4	2	$-0.202\,023\,902\,676\,481 \times 10^{-2}$	28	1	2	$-0.492\,676\,637\,589\,284$
13	−4	4	$0.171\,346\,792\,457\,471 \times 10^{3}$	29	2	0	$0.655\,540\,456\,406\,790 \times 10^{-1}$
14	−4	5	$-0.425\,597\,804\,058\,632 \times 10^{1}$	30	3	2	$-0.240\,462\,535\,078\,530$
15	−3	0	$0.691\,346\,085\,000\,334 \times 10^{-5}$	31	4	0	$-0.269\,798\,180\,310\,075 \times 10^{-1}$
16	−3	1	$0.151\,140\,509\,678\,925 \times 10^{-2}$	32	4	1	$0.128\,369\,435\,967\,012$

Table A1.3. Coefficients and exponents of the backward equation $v_{3c}(p,T)$ for subregion 3c

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	6	$0.311\,967\,788\,763\,030 \times 10^{1}$	19	−2	4	$0.234\,604\,891\,591\,616 \times 10^{3}$
2	−12	8	$0.276\,713\,458\,847\,564 \times 10^{5}$	20	−2	5	$0.377\,515\,668\,966\,951 \times 10^{4}$
3	−12	10	$0.322\,583\,103\,403\,269 \times 10^{8}$	21	−1	0	$0.158\,646\,812\,591\,361 \times 10^{-1}$
4	−10	6	$-0.342\,416\,065\,095\,363 \times 10^{3}$	22	−1	1	$0.707\,906\,336\,241\,843$
5	−10	8	$-0.899\,732\,529\,907\,377 \times 10^{6}$	23	−1	2	$0.126\,016\,225\,146\,570 \times 10^{2}$
6	−10	10	$-0.793\,892\,049\,821\,251 \times 10^{8}$	24	0	0	$0.736\,143\,655\,772\,152$
7	−8	5	$0.953\,193\,003\,217\,388 \times 10^{2}$	25	0	1	$0.676\,544\,268\,999\,101$
8	−8	6	$0.229\,784\,742\,345\,072 \times 10^{4}$	26	0	2	$-0.178\,100\,588\,189\,137 \times 10^{2}$
9	−8	7	$0.175\,336\,675\,322\,499 \times 10^{6}$	27	1	0	$-0.156\,531\,975\,531\,713$
10	−6	8	$0.791\,214\,365\,222\,792 \times 10^{7}$	28	1	2	$0.117\,707\,430\,048\,158 \times 10^{2}$
11	−5	1	$0.319\,933\,345\,844\,209 \times 10^{-4}$	29	2	0	$0.840\,143\,653\,860\,447 \times 10^{-1}$
12	−5	4	$-0.659\,508\,863\,555\,767 \times 10^{2}$	30	2	1	$-0.186\,442\,467\,471\,949$
13	−5	7	$-0.833\,426\,563\,212\,851 \times 10^{6}$	31	2	3	$-0.440\,170\,203\,949\,645 \times 10^{2}$
14	−4	2	$0.645\,734\,680\,583\,292 \times 10^{-1}$	32	2	7	$0.123\,290\,423\,502\,494 \times 10^{7}$
15	−4	8	$-0.382\,031\,020\,570\,813 \times 10^{7}$	33	3	0	$-0.240\,650\,039\,730\,845 \times 10^{-1}$
16	−3	0	$0.406\,398\,848\,470\,079 \times 10^{-4}$	34	3	7	$-0.107\,077\,716\,660\,869 \times 10^{7}$
17	−3	3	$0.310\,327\,498\,492\,008 \times 10^{2}$	35	8	1	$0.438\,319\,858\,566\,475 \times 10^{-1}$
18	−2	0	$-0.892\,996\,718\,483\,724 \times 10^{-3}$				

Table A1.4. Coefficients and exponents of the backward equation $v_{3d}(p,T)$ for subregion 3d

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	4	$-0.452\,484\,847\,171\,645 \times 10^{-9}$	20	−5	1	$-0.436\,701\,347\,922\,356 \times 10^{-5}$
2	−12	6	$0.315\,210\,389\,538\,801 \times 10^{-4}$	21	−5	2	$-0.404\,213\,852\,833\,996 \times 10^{-3}$
3	−12	7	$-0.214\,991\,352\,047\,545 \times 10^{-2}$	22	−5	5	$-0.348\,153\,203\,414\,663 \times 10^{3}$
4	−12	10	$0.508\,058\,874\,808\,345 \times 10^{3}$	23	−5	7	$-0.385\,294\,213\,555\,289 \times 10^{6}$
5	−12	12	$-0.127\,123\,036\,845\,932 \times 10^{8}$	24	−4	0	$0.135\,203\,700\,099\,403 \times 10^{-6}$
6	−12	16	$0.115\,371\,133\,120\,497 \times 10^{13}$	25	−4	1	$0.134\,648\,383\,271\,089 \times 10^{-3}$
7	−10	0	$-0.197\,805\,728\,776\,273 \times 10^{-15}$	26	−4	7	$0.125\,031\,835\,351\,736 \times 10^{6}$
8	−10	2	$0.241\,554\,806\,033\,972 \times 10^{-10}$	27	−3	2	$0.968\,123\,678\,455\,841 \times 10^{-1}$
9	−10	4	$-0.156\,481\,703\,640\,525 \times 10^{-5}$	28	−3	4	$0.225\,660\,517\,512\,438 \times 10^{3}$
10	−10	6	$0.277\,211\,346\,836\,625 \times 10^{-2}$	29	−2	0	$-0.190\,102\,435\,341\,872 \times 10^{-3}$
11	−10	8	$-0.203\,578\,994\,462\,286 \times 10^{2}$	30	−2	1	$-0.299\,628\,410\,819\,229 \times 10^{-1}$
12	−10	10	$0.144\,369\,489\,909\,053 \times 10^{7}$	31	−1	0	$0.500\,833\,915\,372\,121 \times 10^{-2}$
13	−10	14	$-0.411\,254\,217\,946\,539 \times 10^{11}$	32	−1	1	$0.387\,842\,482\,998\,411$
14	−8	3	$0.623\,449\,786\,243\,773 \times 10^{-5}$	33	−1	5	$-0.138\,535\,367\,777\,182 \times 10^{4}$
15	−8	7	$-0.221\,774\,281\,146\,038 \times 10^{2}$	34	0	0	$0.870\,745\,245\,971\,773$
16	−8	8	$-0.689\,315\,087\,933\,158 \times 10^{5}$	35	0	2	$0.171\,946\,252\,068\,742 \times 10^{1}$
17	−8	10	$-0.195\,419\,525\,060\,713 \times 10^{8}$	36	1	0	$-0.326\,650\,121\,426\,383 \times 10^{-1}$
18	−6	6	$0.316\,373\,510\,564\,015 \times 10^{4}$	37	1	6	$0.498\,044\,171\,727\,877 \times 10^{4}$
19	−6	8	$0.224\,040\,754\,426\,988 \times 10^{7}$	38	3	0	$0.551\,478\,022\,765\,087 \times 10^{-2}$

Table A1.5. Coefficients and exponents of the backward equation $v_{3e}(p,T)$ for subregion 3e

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	14	$0.715\,815\,808\,404\,721 \times 10^{9}$	16	−3	6	$0.475\,992\,667\,717\,124 \times 10^{5}$
2	−12	16	$-0.114\,328\,360\,753\,449 \times 10^{12}$	17	−3	7	$-0.266\,627\,750\,390\,341 \times 10^{6}$
3	−10	3	$0.376\,531\,002\,015\,720 \times 10^{-11}$	18	−2	0	$-0.153\,314\,954\,386\,524 \times 10^{-3}$
4	−10	6	$-0.903\,983\,668\,691\,157 \times 10^{-4}$	19	−2	1	$0.305\,638\,404\,828\,265$
5	−10	10	$0.665\,695\,908\,836\,252 \times 10^{6}$	20	−2	3	$0.123\,654\,999\,499\,486 \times 10^{3}$
6	−10	14	$0.535\,364\,174\,960\,127 \times 10^{10}$	21	−2	4	$-0.104\,390\,794\,213\,011 \times 10^{4}$
7	−10	16	$0.794\,977\,402\,335\,603 \times 10^{11}$	22	−1	0	$-0.157\,496\,516\,174\,308 \times 10^{-1}$
8	−8	7	$0.922\,230\,563\,421\,437 \times 10^{2}$	23	0	0	$0.685\,331\,118\,940\,253$
9	−8	8	$-0.142\,586\,073\,991\,215 \times 10^{6}$	24	0	1	$0.178\,373\,462\,873\,903 \times 10^{1}$
10	−8	10	$-0.111\,796\,381\,424\,162 \times 10^{7}$	25	1	0	$-0.544\,674\,124\,878\,910$
11	−6	6	$0.896\,121\,629\,640\,760 \times 10^{4}$	26	1	4	$0.204\,529\,931\,318\,843 \times 10^{4}$
12	−5	6	$-0.669\,989\,239\,070\,491 \times 10^{4}$	27	1	6	$-0.228\,342\,359\,328\,752 \times 10^{5}$
13	−4	2	$0.451\,242\,538\,486\,834 \times 10^{-2}$	28	2	0	$0.413\,197\,481\,515\,899$
14	−4	4	$-0.339\,731\,325\,977\,713 \times 10^{2}$	29	2	2	$-0.341\,931\,835\,910\,405 \times 10^{2}$
15	−3	2	$-0.120\,523\,111\,552\,278 \times 10^{1}$				

Table A1.6. Coefficients and exponents of the backward equation $v_{3f}(p,T)$ for subregion 3f

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	−3	$-0.251\,756\,547\,792\,325 \times 10^{-7}$	22	10	−6	$0.470\,942\,606\,221\,652 \times 10^{-5}$
2	0	−2	$0.601\,307\,193\,668\,763 \times 10^{-5}$	23	12	−10	$0.195\,049\,710\,391\,712 \times 10^{-12}$
3	0	−1	$-0.100\,615\,977\,450\,049 \times 10^{-2}$	24	12	−8	$-0.911\,627\,886\,266\,077 \times 10^{-8}$
4	0	0	$0.999\,969\,140\,252\,192$	25	12	−4	$0.604\,374\,640\,201\,265 \times 10^{-3}$
5	0	1	$0.214\,107\,759\,236\,486 \times 10^{1}$	26	14	−12	$-0.225\,132\,933\,900\,136 \times 10^{-15}$
6	0	2	$-0.165\,175\,571\,959\,086 \times 10^{2}$	27	14	−10	$0.610\,916\,973\,582\,981 \times 10^{-11}$
7	1	−1	$-0.141\,987\,303\,638\,727 \times 10^{-2}$	28	14	−8	$-0.303\,063\,908\,043\,404 \times 10^{-6}$
8	1	1	$0.269\,251\,915\,156\,554 \times 10^{1}$	29	14	−6	$-0.137\,796\,070\,798\,409 \times 10^{-4}$
9	1	2	$0.349\,741\,815\,858\,722 \times 10^{2}$	30	14	−4	$-0.919\,296\,736\,666\,106 \times 10^{-3}$
10	1	3	$-0.300\,208\,695\,771\,783 \times 10^{2}$	31	16	−10	$0.639\,288\,223\,132\,545 \times 10^{-9}$
11	2	0	$-0.131\,546\,288\,252\,539 \times 10^{1}$	32	16	−8	$0.753\,259\,479\,898\,699 \times 10^{-6}$
12	2	1	$-0.839\,091\,277\,286\,169 \times 10^{1}$	33	18	−12	$-0.400\,321\,478\,682\,929 \times 10^{-12}$
13	3	−5	$0.181\,545\,608\,337\,015 \times 10^{-9}$	34	18	−10	$0.756\,140\,294\,351\,614 \times 10^{-8}$
14	3	−2	$-0.591\,099\,206\,478\,909 \times 10^{-3}$	35	20	−12	$-0.912\,082\,054\,034\,891 \times 10^{-11}$
15	3	0	$0.152\,115\,067\,087\,106 \times 10^{1}$	36	20	−10	$-0.237\,612\,381\,140\,539 \times 10^{-7}$
16	4	−3	$0.252\,956\,470\,663\,225 \times 10^{-4}$	37	20	−6	$0.269\,586\,010\,591\,874 \times 10^{-4}$
17	5	−8	$0.100\,726\,265\,203\,786 \times 10^{-14}$	38	22	−12	$-0.732\,828\,135\,157\,839 \times 10^{-10}$
18	5	1	$-0.149\,774\,533\,860\,650 \times 10^{1}$	39	24	−12	$0.241\,995\,578\,306\,660 \times 10^{-9}$
19	6	−6	$-0.793\,940\,970\,562\,969 \times 10^{-9}$	40	24	−4	$-0.405\,735\,532\,730\,322 \times 10^{-3}$
20	7	−4	$-0.150\,290\,891\,264\,717 \times 10^{-3}$	41	28	−12	$0.189\,424\,143\,498\,011 \times 10^{-9}$
21	7	1	$0.151\,205\,531\,275\,133 \times 10^{1}$	42	32	−12	$-0.486\,632\,965\,074\,563 \times 10^{-9}$

Table A1.7. Coefficients and exponents of the backward equation $v_{3g}(p,T)$ for subregion 3g

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	7	$0.412\,209\,020\,652\,996 \times 10^{-4}$	20	−2	3	$-0.910\,782\,540\,134\,681 \times 10^{2}$
2	−12	12	$-0.114\,987\,238\,280\,587 \times 10^{7}$	21	−2	5	$0.135\,033\,227\,281\,565 \times 10^{6}$
3	−12	14	$0.948\,180\,885\,032\,080 \times 10^{10}$	22	−2	14	$-0.712\,949\,383\,408\,211 \times 10^{19}$
4	−12	18	$-0.195\,788\,865\,718\,971 \times 10^{18}$	23	−2	24	$-0.104\,578\,785\,289\,542 \times 10^{37}$
5	−12	22	$0.496\,250\,704\,871\,300 \times 10^{25}$	24	−1	2	$0.304\,331\,584\,444\,093 \times 10^{2}$
6	−12	24	$-0.105\,549\,884\,548\,496 \times 10^{29}$	25	−1	8	$0.593\,250\,797\,959\,445 \times 10^{10}$
7	−10	14	$-0.758\,642\,165\,988\,278 \times 10^{12}$	26	−1	18	$-0.364\,174\,062\,110\,798 \times 10^{28}$
8	−10	20	$-0.922\,172\,769\,596\,101 \times 10^{23}$	27	0	0	$0.921\,791\,403\,532\,461$
9	−10	24	$0.725\,379\,072\,059\,348 \times 10^{30}$	28	0	1	$-0.337\,693\,609\,657\,471$
10	−8	7	$-0.617\,718\,249\,205\,859 \times 10^{2}$	29	0	2	$-0.724\,644\,143\,758\,508 \times 10^{2}$
11	−8	8	$0.107\,555\,033\,344\,858 \times 10^{5}$	30	1	0	$-0.110\,480\,239\,272\,601$
12	−8	10	$-0.379\,545\,802\,336\,487 \times 10^{8}$	31	1	1	$0.536\,516\,031\,875\,059 \times 10^{1}$
13	−8	12	$0.228\,646\,846\,221\,831 \times 10^{12}$	32	1	3	$-0.291\,441\,872\,156\,205 \times 10^{4}$
14	−6	8	$-0.499\,741\,093\,010\,619 \times 10^{7}$	33	3	24	$0.616\,338\,176\,535\,305 \times 10^{40}$
15	−6	22	$-0.280\,214\,310\,054\,101 \times 10^{31}$	34	5	22	$-0.120\,889\,175\,861\,180 \times 10^{39}$
16	−5	7	$0.104\,915\,406\,769\,586 \times 10^{7}$	35	6	12	$0.818\,396\,024\,524\,612 \times 10^{23}$
17	−5	20	$0.613\,754\,229\,168\,619 \times 10^{28}$	36	8	3	$0.940\,781\,944\,835\,829 \times 10^{9}$
18	−4	22	$0.802\,056\,715\,528\,378 \times 10^{32}$	37	10	0	$-0.367\,279\,669\,545\,448 \times 10^{5}$
19	−3	7	$-0.298\,617\,819\,828\,065 \times 10^{8}$	38	10	6	$-0.837\,513\,931\,798\,655 \times 10^{16}$

Table A1.8. Coefficients and exponents of the backward equation $v_{3h}(p,T)$ for subregion 3h

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	8	$0.561\,379\,678\,887\,577 \times 10^{-1}$	16	−6	8	$-0.656\,174\,421\,999\,594 \times 10^{7}$
2	−12	12	$0.774\,135\,421\,587\,083 \times 10^{10}$	17	−5	2	$0.156\,362\,212\,977\,396 \times 10^{-4}$
3	−10	4	$0.111\,482\,975\,877\,938 \times 10^{-8}$	18	−5	3	$-0.212\,946\,257\,021\,400 \times 10^{1}$
4	−10	6	$-0.143\,987\,128\,208\,183 \times 10^{-2}$	19	−5	4	$0.135\,249\,306\,374\,858 \times 10^{2}$
5	−10	8	$0.193\,696\,558\,764\,920 \times 10^{4}$	20	−4	2	$0.177\,189\,164\,145\,813$
6	−10	10	$-0.605\,971\,823\,585\,005 \times 10^{9}$	21	−4	4	$0.139\,499\,167\,345\,464 \times 10^{4}$
7	−10	14	$0.171\,951\,568\,124\,337 \times 10^{14}$	22	−3	1	$-0.703\,670\,932\,036\,388 \times 10^{-2}$
8	−10	16	$-0.185\,461\,154\,985\,145 \times 10^{17}$	23	−3	2	$-0.152\,011\,044\,389\,648$
9	−8	0	$0.387\,851\,168\,078\,010 \times 10^{-16}$	24	−2	0	$0.981\,916\,922\,991\,113 \times 10^{-4}$
10	−8	1	$-0.395\,464\,327\,846\,105 \times 10^{-13}$	25	−1	0	$0.147\,199\,658\,618\,076 \times 10^{-2}$
11	−8	6	$-0.170\,875\,935\,679\,023 \times 10^{3}$	26	−1	2	$0.202\,618\,487\,025\,578 \times 10^{2}$
12	−8	7	$-0.212\,010\,620\,701\,220 \times 10^{4}$	27	0	0	$0.899\,345\,518\,944\,240$
13	−8	8	$0.177\,683\,337\,348\,191 \times 10^{8}$	28	1	0	$-0.211\,346\,402\,240\,858$
14	−6	4	$0.110\,177\,443\,629\,575 \times 10^{2}$	29	1	2	$0.249\,971\,752\,957\,491 \times 10^{2}$
15	−6	6	$-0.234\,396\,091\,693\,313 \times 10^{6}$				

Table A1.9. Coefficients and exponents of the backward equation $v_{3i}(p,T)$ for subregion 3i

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	$0.106\,905\,684\,359\,136 \times 10^{1}$	22	12	−12	$0.164\,395\,334\,345\,040 \times 10^{-23}$
2	0	1	$-0.148\,620\,857\,922\,333 \times 10^{1}$	23	12	−6	$-0.339\,823\,323\,754\,373 \times 10^{-5}$
3	0	10	$0.259\,862\,256\,980\,408 \times 10^{15}$	24	12	−4	$-0.135\,268\,639\,905\,021 \times 10^{-1}$
4	1	−4	$-0.446\,352\,055\,678\,749 \times 10^{-11}$	25	14	−10	$-0.723\,252\,514\,211\,625 \times 10^{-14}$
5	1	−2	$-0.566\,620\,757\,170\,032 \times 10^{-6}$	26	14	−8	$0.184\,386\,437\,538\,366 \times 10^{-8}$
6	1	−1	$-0.235\,302\,885\,736\,849 \times 10^{-2}$	27	14	−4	$-0.463\,959\,533\,752\,385 \times 10^{-1}$
7	1	0	$-0.269\,226\,321\,968\,839$	28	14	5	$-0.992\,263\,100\,376\,750 \times 10^{14}$
8	2	0	$0.922\,024\,992\,944\,392 \times 10^{1}$	29	18	−12	$0.688\,169\,154\,439\,335 \times 10^{-16}$
9	3	−5	$0.357\,633\,505\,503\,772 \times 10^{-11}$	30	18	−10	$-0.222\,620\,998\,452\,197 \times 10^{-10}$
10	3	0	$-0.173\,942\,565\,562\,222 \times 10^{2}$	31	18	−8	$-0.540\,843\,018\,624\,083 \times 10^{-7}$
11	4	−3	$0.700\,681\,785\,556\,229 \times 10^{-5}$	32	18	−6	$0.345\,570\,606\,200\,257 \times 10^{-2}$
12	4	−2	$-0.267\,050\,351\,075\,768 \times 10^{-3}$	33	18	2	$0.422\,275\,800\,304\,086 \times 10^{11}$
13	4	−1	$-0.231\,779\,669\,675\,624 \times 10^{1}$	34	20	−12	$-0.126\,974\,478\,770\,487 \times 10^{-14}$
14	5	−6	$-0.753\,533\,046\,979\,752 \times 10^{-12}$	35	20	−10	$0.927\,237\,985\,153\,679 \times 10^{-9}$
15	5	−1	$0.481\,337\,131\,452\,891 \times 10^{1}$	36	22	−12	$0.612\,670\,812\,016\,489 \times 10^{-13}$
16	5	12	$-0.223\,286\,270\,422\,356 \times 10^{22}$	37	24	−12	$-0.722\,693\,924\,063\,497 \times 10^{-11}$
17	7	−4	$-0.118\,746\,004\,987\,383 \times 10^{-4}$	38	24	−8	$-0.383\,669\,502\,636\,822 \times 10^{-3}$
18	7	−3	$0.646\,412\,934\,136\,496 \times 10^{-2}$	39	32	−10	$0.374\,684\,572\,410\,204 \times 10^{-3}$
19	8	−6	$-0.410\,588\,536\,330\,937 \times 10^{-9}$	40	32	−5	$-0.931\,976\,897\,511\,086 \times 10^{5}$
20	8	10	$0.422\,739\,537\,057\,241 \times 10^{20}$	41	36	−10	$-0.247\,690\,616\,026\,922 \times 10^{-1}$
21	10	−8	$0.313\,698\,180\,473\,812 \times 10^{-12}$	42	36	−8	$0.658\,110\,546\,759\,474 \times 10^{2}$

Table A1.10. Coefficients and exponents of the backward equation $v_{3j}(p,T)$ for subregion 3j

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	−1	$-0.111\,371\,317\,395\,540 \times 10^{-3}$	16	10	−6	$-0.960\,754\,116\,701\,669 \times 10^{-8}$
2	0	0	$0.100\,342\,892\,423\,685 \times 10^{1}$	17	12	−8	$-0.510\,572\,269\,720\,488 \times 10^{-10}$
3	0	1	$0.530\,615\,581\,928\,979 \times 10^{1}$	18	12	−3	$0.767\,373\,781\,404\,211 \times 10^{-2}$
4	1	−2	$0.179\,058\,760\,078\,792 \times 10^{-5}$	19	14	−10	$0.663\,855\,469\,485\,254 \times 10^{-14}$
5	1	−1	$-0.728\,541\,958\,464\,774 \times 10^{-3}$	20	14	−8	$-0.717\,590\,735\,526\,745 \times 10^{-9}$
6	1	1	$-0.187\,576\,133\,371\,704 \times 10^{2}$	21	14	−5	$0.146\,564\,542\,926\,508 \times 10^{-4}$
7	2	−1	$0.199\,060\,874\,071\,849 \times 10^{-2}$	22	16	−10	$0.309\,029\,474\,277\,013 \times 10^{-11}$
8	2	1	$0.243\,574\,755\,377\,290 \times 10^{2}$	23	18	−12	$-0.464\,216\,300\,971\,708 \times 10^{-15}$
9	3	−2	$-0.177\,040\,785\,499\,444 \times 10^{-3}$	24	20	−12	$-0.390\,499\,637\,961\,161 \times 10^{-13}$
10	4	−2	$-0.259\,680\,385\,227\,130 \times 10^{-2}$	25	20	−10	$-0.236\,716\,126\,781\,431 \times 10^{-9}$
11	4	2	$-0.198\,704\,578\,406\,823 \times 10^{3}$	26	24	−12	$0.454\,652\,854\,268\,717 \times 10^{-11}$
12	5	−3	$0.738\,627\,790\,224\,287 \times 10^{-4}$	27	24	−6	$-0.422\,271\,787\,482\,497 \times 10^{-2}$
13	5	−2	$-0.236\,264\,692\,844\,138 \times 10^{-2}$	28	28	−12	$0.283\,911\,742\,354\,706 \times 10^{-10}$
14	5	0	$-0.161\,023\,121\,314\,333 \times 10^{1}$	29	28	−5	$0.270\,929\,002\,720\,228 \times 10^{1}$
15	6	3	$0.622\,322\,971\,786\,473 \times 10^{4}$				

Table A1.11. Coefficients and exponents of the backward equation $v_{3k}(p,T)$ for subregion 3k

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−2	10	$-0.401\,215\,699\,576\,099 \times 10^{9}$	18	1	2	$-0.194\,646\,110\,037\,079 \times 10^{3}$
2	−2	12	$0.484\,501\,478\,318\,406 \times 10^{11}$	19	2	−8	$0.808\,354\,639\,772\,825 \times 10^{-15}$
3	−1	−5	$0.394\,721\,471\,363\,678 \times 10^{-14}$	20	2	−6	$-0.180\,845\,209\,145\,470 \times 10^{-10}$
4	−1	6	$0.372\,629\,967\,374\,147 \times 10^{5}$	21	2	−3	$-0.696\,664\,158\,132\,412 \times 10^{-5}$
5	0	−12	$-0.369\,794\,374\,168\,666 \times 10^{-29}$	22	2	−2	$-0.181\,057\,560\,300\,994 \times 10^{-2}$
6	0	−6	$-0.380\,436\,407\,012\,452 \times 10^{-14}$	23	2	0	$0.255\,830\,298\,579\,027 \times 10^{1}$
7	0	−2	$0.475\,361\,629\,970\,233 \times 10^{-6}$	24	2	4	$0.328\,913\,873\,658\,481 \times 10^{4}$
8	0	−1	$-0.879\,148\,916\,140\,706 \times 10^{-3}$	25	5	−12	$-0.173\,270\,241\,249\,904 \times 10^{-18}$
9	0	0	$0.844\,317\,863\,844\,331$	26	5	−6	$-0.661\,876\,792\,558\,034 \times 10^{-6}$
10	0	1	$0.122\,433\,162\,656\,600 \times 10^{2}$	27	5	−3	$-0.395\,688\,923\,421\,250 \times 10^{-2}$
11	0	2	$-0.104\,529\,634\,830\,279 \times 10^{3}$	28	6	−12	$0.604\,203\,299\,819\,132 \times 10^{-17}$
12	0	3	$0.589\,702\,771\,277\,429 \times 10^{3}$	29	6	−10	$-0.400\,879\,935\,920\,517 \times 10^{-13}$
13	0	14	$-0.291\,026\,851\,164\,444 \times 10^{14}$	30	6	−8	$0.160\,751\,107\,464\,958 \times 10^{-8}$
14	1	−3	$0.170\,343\,072\,841\,850 \times 10^{-5}$	31	6	−5	$0.383\,719\,409\,025\,556 \times 10^{-4}$
15	1	−2	$-0.277\,617\,606\,975\,748 \times 10^{-3}$	32	8	−12	$-0.649\,565\,446\,702\,457 \times 10^{-14}$
16	1	0	$-0.344\,709\,605\,486\,686 \times 10^{1}$	33	10	−12	$-0.149\,095\,328\,506\,000 \times 10^{-11}$
17	1	1	$0.221\,333\,862\,447\,095 \times 10^{2}$	34	12	−10	$0.541\,449\,377\,329\,581 \times 10^{-8}$

Table A1.12. Coefficients and exponents of the backward equation $v_{3l}(p,T)$ for subregion 3l

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	14	$0.260\,702\,058\,647\,537 \times 10^{10}$	23	−3	20	$-0.695\,953\,622\,348\,829 \times 10^{33}$
2	−12	16	$-0.188\,277\,213\,604\,704 \times 10^{15}$	24	−2	2	$0.110\,609\,027\,472\,280$
3	−12	18	$0.554\,923\,870\,289\,667 \times 10^{19}$	25	−2	3	$0.721\,559\,163\,361\,354 \times 10^{2}$
4	−12	20	$-0.758\,966\,946\,387\,758 \times 10^{23}$	26	−2	10	$-0.306\,367\,307\,532\,219 \times 10^{15}$
5	−12	22	$0.413\,865\,186\,848\,908 \times 10^{27}$	27	−1	0	$0.265\,839\,618\,885\,530 \times 10^{-4}$
6	−10	14	$-0.815\,038\,000\,738\,060 \times 10^{12}$	28	−1	1	$0.253\,392\,392\,889\,754 \times 10^{-1}$
7	−10	24	$-0.381\,458\,260\,489\,955 \times 10^{33}$	29	−1	3	$-0.214\,443\,041\,836\,579 \times 10^{3}$
8	−8	6	$-0.123\,239\,564\,600\,519 \times 10^{-1}$	30	0	0	$0.937\,846\,601\,489\,667$
9	−8	10	$0.226\,095\,631\,437\,174 \times 10^{8}$	31	0	1	$0.223\,184\,043\,101\,700 \times 10^{1}$
10	−8	12	$-0.495\,017\,809\,506\,720 \times 10^{12}$	32	0	2	$0.338\,401\,222\,509\,191 \times 10^{2}$
11	−8	14	$0.529\,482\,996\,422\,863 \times 10^{16}$	33	0	12	$0.494\,237\,237\,179\,718 \times 10^{21}$
12	−8	18	$-0.444\,359\,478\,746\,295 \times 10^{23}$	34	1	0	$-0.198\,068\,404\,154\,428$
13	−8	24	$0.521\,635\,864\,527\,315 \times 10^{35}$	35	1	16	$-0.141\,415\,349\,881\,140 \times 10^{31}$
14	−8	36	$-0.487\,095\,672\,740\,742 \times 10^{55}$	36	2	1	$-0.993\,862\,421\,613\,651 \times 10^{2}$
15	−6	8	$-0.714\,430\,209\,937\,547 \times 10^{6}$	37	4	0	$0.125\,070\,534\,142\,731 \times 10^{3}$
16	−5	4	$0.127\,868\,634\,615\,495$	38	5	0	$-0.996\,473\,529\,004\,439 \times 10^{3}$
17	−5	5	$-0.100\,752\,127\,917\,598 \times 10^{2}$	39	5	1	$0.473\,137\,909\,872\,765 \times 10^{5}$
18	−4	7	$0.777\,451\,437\,960\,990 \times 10^{7}$	40	6	14	$0.116\,662\,121\,219\,322 \times 10^{33}$
19	−4	16	$-0.108\,105\,480\,796\,471 \times 10^{25}$	41	10	4	$-0.315\,874\,976\,271\,533 \times 10^{16}$
20	−3	1	$-0.357\,578\,581\,169\,659 \times 10^{-5}$	42	10	12	$-0.445\,703\,369\,196\,945 \times 10^{33}$
21	−3	3	$-0.212\,857\,169\,423\,484 \times 10^{1}$	43	14	10	$0.642\,794\,932\,373\,694 \times 10^{33}$
22	−3	18	$0.270\,706\,111\,085\,238 \times 10^{30}$				

Table A1.13. Coefficients and exponents of the backward equation $v_{3m}(p,T)$ for subregion 3m

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	0.811 384 363 481 847	21	28	20	$0.368\,193\,926\,183\,570 \times 10^{60}$
2	3	0	$-0.568\,199\,310\,990\,094 \times 10^{4}$	22	2	22	$0.170\,215\,539\,458\,936 \times 10^{18}$
3	8	0	$-0.178\,657\,198\,172\,556 \times 10^{11}$	23	16	22	$0.639\,234\,909\,918\,741 \times 10^{42}$
4	20	2	$0.795\,537\,657\,613\,427 \times 10^{32}$	24	0	24	$-0.821\,698\,160\,721\,956 \times 10^{15}$
5	1	5	$-0.814\,568\,209\,346\,872 \times 10^{5}$	25	5	24	$-0.795\,260\,241\,872\,306 \times 10^{24}$
6	3	5	$-0.659\,774\,567\,602\,874 \times 10^{8}$	26	0	28	$0.233\,415\,869\,478\,510 \times 10^{18}$
7	4	5	$-0.152\,861\,148\,659\,302 \times 10^{11}$	27	3	28	$-0.600\,079\,934\,586\,803 \times 10^{23}$
8	5	5	$-0.560\,165\,667\,510\,446 \times 10^{12}$	28	4	28	$0.594\,584\,382\,273\,384 \times 10^{25}$
9	1	6	$0.458\,384\,828\,593\,949 \times 10^{6}$	29	12	28	$0.189\,461\,279\,349\,492 \times 10^{40}$
10	6	6	$-0.385\,754\,000\,383\,848 \times 10^{14}$	30	16	28	$-0.810\,093\,428\,842\,645 \times 10^{46}$
11	2	7	$0.453\,735\,800\,004\,273 \times 10^{8}$	31	1	32	$0.188\,813\,911\,076\,809 \times 10^{22}$
12	4	8	$0.939\,454\,935\,735\,563 \times 10^{12}$	32	8	32	$0.111\,052\,244\,098\,768 \times 10^{36}$
13	14	8	$0.266\,572\,856\,432\,938 \times 10^{28}$	33	14	32	$0.291\,133\,958\,602\,503 \times 10^{46}$
14	2	10	$-0.547\,578\,313\,899\,097 \times 10^{10}$	34	0	36	$-0.329\,421\,923\,951\,460 \times 10^{22}$
15	5	10	$0.200\,725\,701\,112\,386 \times 10^{15}$	35	2	36	$-0.137\,570\,282\,536\,696 \times 10^{26}$
16	3	12	$0.185\,007\,245\,563\,239 \times 10^{13}$	36	3	36	$0.181\,508\,996\,303\,902 \times 10^{28}$
17	0	14	$0.185\,135\,446\,828\,337 \times 10^{9}$	37	4	36	$-0.346\,865\,122\,768\,353 \times 10^{30}$
18	1	14	$-0.170\,451\,090\,076\,385 \times 10^{12}$	38	8	36	$-0.211\,961\,148\,774\,260 \times 10^{38}$
19	1	18	$0.157\,890\,366\,037\,614 \times 10^{15}$	39	14	36	$-0.128\,617\,899\,887\,675 \times 10^{49}$
20	1	20	$-0.202\,530\,509\,748\,774 \times 10^{16}$	40	24	36	$0.479\,817\,895\,699\,239 \times 10^{65}$

Table A1.14. Coefficients and exponents of the backward equation $v_{3n}(p,T)$ for subregion 3n

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	−12	$0.280\,967\,799\,943\,151 \times 10^{-38}$	21	3	−6	$0.705\,412\,100\,773\,699 \times 10^{-11}$
2	3	−12	$0.614\,869\,006\,573\,609 \times 10^{-30}$	22	4	−6	$0.258\,585\,887\,897\,486 \times 10^{-8}$
3	4	−12	$0.582\,238\,667\,048\,942 \times 10^{-27}$	23	2	−5	$-0.493\,111\,362\,030\,162 \times 10^{-10}$
4	6	−12	$0.390\,628\,369\,238\,462 \times 10^{-22}$	24	4	−5	$-0.158\,649\,699\,894\,543 \times 10^{-5}$
5	7	−12	$0.821\,445\,758\,255\,119 \times 10^{-20}$	25	7	−5	−0.525 037 427 886 100
6	10	−12	$0.402\,137\,961\,842\,776 \times 10^{-14}$	26	4	−4	$0.220\,019\,901\,729\,615 \times 10^{-2}$
7	12	−12	$0.651\,718\,171\,878\,301 \times 10^{-12}$	27	3	−3	$-0.643\,064\,132\,636\,925 \times 10^{-2}$
8	14	−12	$-0.211\,773\,355\,803\,058 \times 10^{-7}$	28	5	−3	$0.629\,154\,149\,015\,048 \times 10^{2}$
9	18	−12	$0.264\,953\,354\,380\,072 \times 10^{-2}$	29	6	−3	$0.135\,147\,318\,617\,061 \times 10^{3}$
10	0	−10	$-0.135\,031\,446\,451\,331 \times 10^{-31}$	30	0	−2	$0.240\,560\,808\,321\,713 \times 10^{-6}$
11	3	−10	$-0.607\,246\,643\,970\,893 \times 10^{-23}$	31	0	−1	$-0.890\,763\,306\,701\,305 \times 10^{-3}$
12	5	−10	$-0.402\,352\,115\,234\,494 \times 10^{-18}$	32	3	−1	$-0.440\,209\,599\,407\,714 \times 10^{4}$
13	6	−10	$-0.744\,938\,506\,925\,544 \times 10^{-16}$	33	1	0	$-0.302\,807\,107\,747\,776 \times 10^{3}$
14	8	−10	$0.189\,917\,206\,526\,237 \times 10^{-12}$	34	0	1	$0.159\,158\,748\,314\,599 \times 10^{4}$
15	12	−10	$0.364\,975\,183\,508\,473 \times 10^{-5}$	35	1	1	$0.232\,534\,272\,709\,876 \times 10^{6}$
16	0	−8	$0.177\,274\,872\,361\,946 \times 10^{-25}$	36	0	2	$-0.792\,681\,207\,132\,600 \times 10^{6}$
17	3	−8	$-0.334\,952\,758\,812\,999 \times 10^{-18}$	37	1	4	$-0.869\,871\,364\,662\,769 \times 10^{11}$
18	7	−8	$-0.421\,537\,726\,098\,389 \times 10^{-8}$	38	0	5	$0.354\,542\,769\,185\,671 \times 10^{12}$
19	12	−8	$-0.391\,048\,167\,929\,649 \times 10^{-1}$	39	1	6	$0.400\,849\,240\,129\,329 \times 10^{15}$
20	2	−6	$0.541\,276\,911\,564\,176 \times 10^{-13}$				

Table A1.15. Coefficients and exponents of the backward equation $v_{3o}(p,T)$ for subregion 3o

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	−12	$0.128\,746\,023\,979\,718 \times 10^{-34}$	13	6	−8	$0.814\,897\,605\,805\,513 \times 10^{-14}$
2	0	−4	$-0.735\,234\,770\,382\,342 \times 10^{-11}$	14	7	−12	$0.425\,596\,631\,351\,839 \times 10^{-25}$
3	0	−1	$0.289\,078\,692\,149\,150 \times 10^{-2}$	15	8	−10	$-0.387\,449\,113\,787\,755 \times 10^{-17}$
4	2	−1	$0.244\,482\,731\,907\,223$	16	8	−8	$0.139\,814\,747\,930\,240 \times 10^{-12}$
5	3	−10	$0.141\,733\,492\,030\,985 \times 10^{-23}$	17	8	−4	$-0.171\,849\,638\,951\,521 \times 10^{-2}$
6	4	−12	$-0.354\,533\,853\,059\,476 \times 10^{-28}$	18	10	−12	$0.641\,890\,529\,513\,296 \times 10^{-21}$
7	4	−8	$-0.594\,539\,202\,901\,431 \times 10^{-17}$	19	10	−8	$0.118\,960\,578\,072\,018 \times 10^{-10}$
8	4	−5	$-0.585\,188\,401\,782\,779 \times 10^{-8}$	20	14	−12	$-0.155\,282\,762\,571\,611 \times 10^{-17}$
9	4	−4	$0.201\,377\,325\,411\,803 \times 10^{-5}$	21	14	−8	$0.233\,907\,907\,347\,507 \times 10^{-7}$
10	4	−1	$0.138\,647\,388\,209\,306 \times 10^{1}$	22	20	−12	$-0.174\,093\,247\,766\,213 \times 10^{-12}$
11	5	−4	$-0.173\,959\,365\,084\,772 \times 10^{-4}$	23	20	−10	$0.377\,682\,649\,089\,149 \times 10^{-8}$
12	5	−3	$0.137\,680\,878\,349\,369 \times 10^{-2}$	24	24	−12	$-0.516\,720\,236\,575\,302 \times 10^{-10}$

Table A1.16. Coefficients and exponents of the backward equation $v_{3p}(p,T)$ for subregion 3p

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	−1	$-0.982\,825\,342\,010\,366 \times 10^{-4}$	15	12	−12	$0.343\,480\,022\,104\,968 \times 10^{-25}$
2	0	0	$0.105\,145\,700\,850\,612 \times 10^{1}$	16	12	−6	$0.816\,256\,095\,947\,021 \times 10^{-5}$
3	0	1	$0.116\,033\,094\,095\,084 \times 10^{3}$	17	12	−5	$0.294\,985\,697\,916\,798 \times 10^{-2}$
4	0	2	$0.324\,664\,750\,281\,543 \times 10^{4}$	18	14	−10	$0.711\,730\,466\,276\,584 \times 10^{-16}$
5	1	1	$-0.123\,592\,348\,610\,137 \times 10^{4}$	19	14	−8	$0.400\,954\,763\,806\,941 \times 10^{-9}$
6	2	−1	$-0.561\,403\,450\,013\,495 \times 10^{-1}$	20	14	−3	$0.107\,766\,027\,032\,853 \times 10^{2}$
7	3	−3	$0.856\,677\,401\,640\,869 \times 10^{-7}$	21	16	−8	$-0.409\,449\,599\,138\,182 \times 10^{-6}$
8	3	0	$0.236\,313\,425\,393\,924 \times 10^{3}$	22	18	−8	$-0.729\,121\,307\,758\,902 \times 10^{-5}$
9	4	−2	$0.972\,503\,292\,350\,109 \times 10^{-2}$	23	20	−10	$0.677\,107\,970\,938\,909 \times 10^{-8}$
10	6	−2	$-0.103\,001\,994\,531\,927 \times 10^{1}$	24	22	−10	$0.602\,745\,973\,022\,975 \times 10^{-7}$
11	7	−5	$-0.149\,653\,706\,199\,162 \times 10^{-8}$	25	24	−12	$-0.382\,323\,011\,855\,257 \times 10^{-10}$
12	7	−4	$-0.215\,743\,778\,861\,592 \times 10^{-4}$	26	24	−8	$0.179\,946\,628\,317\,437 \times 10^{-2}$
13	8	−2	$-0.834\,452\,198\,291\,445 \times 10^{1}$	27	36	−12	$-0.345\,042\,834\,640\,005 \times 10^{-3}$
14	10	−3	$0.586\,602\,660\,564\,988$				

Table A1.17. Coefficients and exponents of the backward equation $v_{3q}(p,T)$ for subregion 3q

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	10	$-0.820\,433\,843\,259\,950 \times 10^{5}$	13	−3	3	$0.232\,808\,472\,983\,776 \times 10^{3}$
2	−12	12	$0.473\,271\,518\,461\,586 \times 10^{11}$	14	−2	0	$-0.142\,808\,220\,416\,837 \times 10^{-4}$
3	−10	6	$-0.805\,950\,021\,005\,413 \times 10^{-1}$	15	−2	1	$-0.643\,596\,060\,678\,456 \times 10^{-2}$
4	−10	7	$0.328\,600\,025\,435\,980 \times 10^{2}$	16	−2	2	$-0.428\,577\,227\,475\,614 \times 10^{1}$
5	−10	8	$-0.356\,617\,029\,982\,490 \times 10^{4}$	17	−2	4	$0.225\,689\,939\,161\,918 \times 10^{4}$
6	−10	10	$-0.172\,985\,781\,433\,335 \times 10^{10}$	18	−1	0	$0.100\,355\,651\,721\,510 \times 10^{-2}$
7	−8	8	$0.351\,769\,232\,729\,192 \times 10^{8}$	19	−1	1	$0.333\,491\,455\,143\,516$
8	−6	6	$-0.775\,489\,259\,985\,144 \times 10^{6}$	20	−1	2	$0.109\,697\,576\,888\,873 \times 10^{1}$
9	−5	2	$0.710\,346\,691\,966\,018 \times 10^{-4}$	21	0	0	$0.961\,917\,379\,376\,452$
10	−5	5	$0.993\,499\,883\,820\,274 \times 10^{5}$	22	1	0	$-0.838\,165\,632\,204\,598 \times 10^{-1}$
11	−4	3	$-0.642\,094\,171\,904\,570$	23	1	1	$0.247\,795\,908\,411\,492 \times 10^{1}$
12	−4	4	$-0.612\,842\,816\,820\,083 \times 10^{4}$	24	1	3	$-0.319\,114\,969\,006\,533 \times 10^{4}$

Table A1.18. Coefficients and exponents of the backward equation $v_{3r}(p,T)$ for subregion 3r

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−8	6	$0.144\,165\,955\,660\,863 \times 10^{-2}$	15	8	−10	$0.399\,988\,795\,693\,162 \times 10^{-12}$
2	−8	14	$-0.701\,438\,599\,628\,258 \times 10^{13}$	16	8	−8	$-0.536\,479\,560\,201\,811 \times 10^{-6}$
3	−3	−3	$-0.830\,946\,716\,459\,219 \times 10^{-16}$	17	8	−5	$0.159\,536\,722\,411\,202 \times 10^{-1}$
4	−3	3	$0.261\,975\,135\,368\,109$	18	10	−12	$0.270\,303\,248\,860\,217 \times 10^{-14}$
5	−3	4	$0.393\,097\,214\,706\,245 \times 10^{3}$	19	10	−10	$0.244\,247\,453\,858\,506 \times 10^{-7}$
6	−3	5	$-0.104\,334\,030\,654\,021 \times 10^{5}$	20	10	−8	$-0.983\,430\,636\,716\,454 \times 10^{-5}$
7	−3	8	$0.490\,112\,654\,154\,211 \times 10^{9}$	21	10	−6	$0.663\,513\,144\,224\,454 \times 10^{-1}$
8	0	−1	$-0.147\,104\,222\,772\,069 \times 10^{-3}$	22	10	−5	$-0.993\,456\,957\,845\,006 \times 10^{1}$
9	0	0	$0.103\,602\,748\,043\,408 \times 10^{1}$	23	10	−4	$0.546\,491\,323\,528\,491 \times 10^{3}$
10	0	1	$0.305\,308\,890\,065\,089 \times 10^{1}$	24	10	−3	$-0.143\,365\,406\,393\,758 \times 10^{5}$
11	0	5	$-0.399\,745\,276\,971\,264 \times 10^{7}$	25	10	−2	$0.150\,764\,974\,125\,511 \times 10^{6}$
12	3	−6	$0.569\,233\,719\,593\,750 \times 10^{-11}$	26	12	−12	$-0.337\,209\,709\,340\,105 \times 10^{-9}$
13	3	−2	$-0.464\,923\,504\,407\,778 \times 10^{-1}$	27	14	−12	$0.377\,501\,980\,025\,469 \times 10^{-8}$
14	8	−12	$-0.535\,400\,396\,512\,906 \times 10^{-17}$				

Table A1.19. Coefficients and exponents of the backward equation $v_{3s}(p,T)$ for subregion 3s

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	20	$-0.532\,466\,612\,140\,254 \times 10^{23}$	16	0	0	$0.965\,961\,650\,599\,775$
2	−12	24	$0.100\,415\,480\,000\,824 \times 10^{32}$	17	0	1	$0.294\,885\,696\,802\,488 \times 10^{1}$
3	−10	22	$-0.191\,540\,001\,821\,367 \times 10^{30}$	18	0	4	$-0.653\,915\,627\,346\,115 \times 10^{5}$
4	−8	14	$0.105\,618\,377\,808\,847 \times 10^{17}$	19	0	28	$0.604\,012\,200\,163\,444 \times 10^{50}$
5	−6	36	$0.202\,281\,884\,477\,061 \times 10^{59}$	20	1	0	$-0.198\,339\,358\,557\,937$
6	−5	8	$0.884\,585\,472\,596\,134 \times 10^{8}$	21	1	32	$-0.175\,984\,090\,163\,501 \times 10^{58}$
7	−5	16	$0.166\,540\,181\,638\,363 \times 10^{23}$	22	3	0	$0.356\,314\,881\,403\,987 \times 10^{1}$
8	−4	6	$-0.313\,563\,197\,669\,111 \times 10^{6}$	23	3	1	$-0.575\,991\,255\,144\,384 \times 10^{3}$
9	−4	32	$-0.185\,662\,327\,545\,324 \times 10^{54}$	24	3	2	$0.456\,213\,415\,338\,071 \times 10^{5}$
10	−3	3	$-0.624\,942\,093\,918\,942 \times 10^{-1}$	25	4	3	$-0.109\,174\,044\,987\,829 \times 10^{8}$
11	−3	8	$-0.504\,160\,724\,132\,590 \times 10^{10}$	26	4	18	$0.437\,796\,099\,975\,134 \times 10^{34}$
12	−2	4	$0.187\,514\,491\,833\,092 \times 10^{5}$	27	4	24	$-0.616\,552\,611\,135\,792 \times 10^{46}$
13	−1	1	$0.121\,399\,979\,993\,217 \times 10^{-2}$	28	5	4	$0.193\,568\,768\,917\,797 \times 10^{10}$
14	−1	2	$0.188\,317\,043\,049\,455 \times 10^{1}$	29	14	24	$0.950\,898\,170\,425\,042 \times 10^{54}$
15	−1	3	$-0.167\,073\,503\,962\,060 \times 10^{4}$				

Table A1.20. Coefficients and exponents of the backward equation $v_{3t}(p,T)$ for subregion 3t

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	$0.155\,287\,249\,586\,268 \times 10^{1}$	18	7	36	$-0.341\,552\,040\,860\,644 \times 10^{51}$
2	0	1	$0.664\,235\,115\,009\,031 \times 10^{1}$	19	10	10	$-0.527\,251\,339\,709\,047 \times 10^{21}$
3	0	4	$-0.289\,366\,236\,727\,210 \times 10^{4}$	20	10	12	$0.245\,375\,640\,937\,055 \times 10^{24}$
4	0	12	$-0.385\,923\,202\,309\,848 \times 10^{13}$	21	10	14	$-0.168\,776\,617\,209\,269 \times 10^{27}$
5	1	0	$-0.291\,002\,915\,783\,761 \times 10^{1}$	22	10	16	$0.358\,958\,955\,867\,578 \times 10^{29}$
6	1	10	$-0.829\,088\,246\,858\,083 \times 10^{12}$	23	10	22	$-0.656\,475\,280\,339\,411 \times 10^{36}$
7	2	0	$0.176\,814\,899\,675\,218 \times 10^{1}$	24	18	18	$0.355\,286\,045\,512\,301 \times 10^{39}$
8	2	6	$-0.534\,686\,695\,713\,469 \times 10^{9}$	25	20	32	$0.569\,021\,454\,413\,270 \times 10^{58}$
9	2	14	$0.160\,464\,608\,687\,834 \times 10^{18}$	26	22	22	$-0.700\,584\,546\,433\,113 \times 10^{48}$
10	3	3	$0.196\,435\,366\,560\,186 \times 10^{6}$	27	22	36	$-0.705\,772\,623\,326\,374 \times 10^{65}$
11	3	8	$0.156\,637\,427\,541\,729 \times 10^{13}$	28	24	24	$0.166\,861\,176\,200\,148 \times 10^{53}$
12	4	0	$-0.178\,154\,560\,260\,006 \times 10^{1}$	29	28	28	$-0.300\,475\,129\,680\,486 \times 10^{61}$
13	4	10	$-0.229\,746\,237\,623\,692 \times 10^{16}$	30	32	22	$-0.668\,481\,295\,196\,808 \times 10^{51}$
14	7	3	$0.385\,659\,001\,648\,006 \times 10^{8}$	31	32	32	$0.428\,432\,338\,620\,678 \times 10^{69}$
15	7	4	$0.110\,554\,446\,790\,543 \times 10^{10}$	32	32	36	$-0.444\,227\,367\,758\,304 \times 10^{72}$
16	7	7	$-0.677\,073\,830\,687\,349 \times 10^{14}$	33	36	36	$-0.281\,396\,013\,562\,745 \times 10^{77}$
17	7	20	$-0.327\,910\,592\,086\,523 \times 10^{31}$				

A2 Coefficients for Auxiliary Equations

Table A2.1. Coefficients and exponents of the auxiliary equation $v_{3u}(p,T)$ for subregion 3u

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	14	$0.122\,088\,349\,258\,355 \times 10^{18}$	20	1	−2	$0.105\,581\,745\,346\,187 \times 10^{-2}$
2	−10	10	$0.104\,216\,468\,608\,488 \times 10^{10}$	21	2	5	$-0.651\,903\,203\,602\,581 \times 10^{15}$
3	−10	12	$-0.882\,666\,931\,564\,652 \times 10^{16}$	22	2	10	$-0.160\,116\,813\,274\,676 \times 10^{25}$
4	−10	14	$0.259\,929\,510\,849\,499 \times 10^{20}$	23	3	−5	$-0.510\,254\,294\,237\,837 \times 10^{-8}$
5	−8	10	$0.222\,612\,779\,142\,211 \times 10^{15}$	24	5	−4	$-0.152\,355\,388\,953\,402$
6	−8	12	$-0.878\,473\,585\,050\,085 \times 10^{18}$	25	5	2	$0.677\,143\,292\,290\,144 \times 10^{12}$
7	−8	14	$-0.314\,432\,577\,551\,552 \times 10^{22}$	26	5	3	$0.276\,378\,438\,378\,930 \times 10^{15}$
8	−6	8	$-0.216\,934\,916\,996\,285 \times 10^{13}$	27	6	−5	$0.116\,862\,983\,141\,686 \times 10^{-1}$
9	−6	12	$0.159\,079\,648\,196\,849 \times 10^{21}$	28	6	2	$-0.301\,426\,947\,980\,171 \times 10^{14}$
10	−5	4	$-0.339\,567\,617\,303\,423 \times 10^{3}$	29	8	−8	$0.169\,719\,813\,884\,840 \times 10^{-7}$
11	−5	8	$0.884\,387\,651\,337\,836 \times 10^{13}$	30	8	8	$0.104\,674\,840\,020\,929 \times 10^{27}$
12	−5	12	$-0.843\,405\,926\,846\,418 \times 10^{21}$	31	10	−4	$-0.108\,016\,904\,560\,140 \times 10^{5}$
13	−3	2	$0.114\,178\,193\,518\,022 \times 10^{2}$	32	12	−12	$-0.990\,623\,601\,934\,295 \times 10^{-12}$
14	−1	−1	$-0.122\,708\,229\,235\,641 \times 10^{-3}$	33	12	−4	$0.536\,116\,483\,602\,738 \times 10^{7}$
15	−1	1	$-0.106\,201\,671\,767\,107 \times 10^{3}$	34	12	4	$0.226\,145\,963\,747\,881 \times 10^{22}$
16	−1	12	$0.903\,443\,213\,959\,313 \times 10^{25}$	35	14	−12	$-0.488\,731\,565\,776\,210 \times 10^{-9}$
17	−1	14	$-0.693\,996\,270\,370\,852 \times 10^{28}$	36	14	−10	$0.151\,001\,548\,880\,670 \times 10^{-4}$
18	0	−3	$0.648\,916\,718\,965\,575 \times 10^{-8}$	37	14	−6	$-0.227\,700\,464\,643\,920 \times 10^{5}$
19	0	1	$0.718\,957\,567\,127\,851 \times 10^{4}$	38	14	6	$-0.781\,754\,507\,698\,846 \times 10^{28}$

Table A2.2. Coefficients and exponents of the auxiliary equation $v_{3v}(p,T)$ for subregion 3v

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−10	−8	$-0.415\,652\,812\,061\,591 \times 10^{-54}$	21	−3	12	$0.742\,705\,723\,302\,738 \times 10^{27}$
2	−8	−12	$0.177\,441\,742\,924\,043 \times 10^{-60}$	22	−2	2	$-0.517\,429\,682\,450\,605 \times 10^{2}$
3	−6	−12	$-0.357\,078\,668\,203\,377 \times 10^{-54}$	23	−2	4	$0.820\,612\,048\,645\,469 \times 10^{7}$
4	−6	−3	$0.359\,252\,213\,604\,114 \times 10^{-25}$	24	−1	−2	$-0.188\,214\,882\,341\,448 \times 10^{-8}$
5	−6	5	$-0.259\,123\,736\,380\,269 \times 10^{2}$	25	−1	0	$0.184\,587\,261\,114\,837 \times 10^{-1}$
6	−6	6	$0.594\,619\,766\,193\,460 \times 10^{5}$	26	0	−2	$-0.135\,830\,407\,782\,663 \times 10^{-5}$
7	−6	8	$-0.624\,184\,007\,103\,158 \times 10^{11}$	27	0	6	$-0.723\,681\,885\,626\,348 \times 10^{17}$
8	−6	10	$0.313\,080\,299\,915\,944 \times 10^{17}$	28	0	10	$-0.223\,449\,194\,054\,124 \times 10^{27}$
9	−5	1	$0.105\,006\,446\,192\,036 \times 10^{-8}$	29	1	−12	$-0.111\,526\,741\,826\,431 \times 10^{-34}$
10	−5	2	$-0.192\,824\,336\,984\,852 \times 10^{-5}$	30	1	−10	$0.276\,032\,601\,145\,151 \times 10^{-28}$
11	−5	6	$0.654\,144\,373\,749\,937 \times 10^{6}$	31	3	3	$0.134\,856\,491\,567\,853 \times 10^{15}$
12	−5	8	$0.513\,117\,462\,865\,044 \times 10^{13}$	32	4	−6	$0.652\,440\,293\,345\,860 \times 10^{-9}$
13	−5	10	$-0.697\,595\,750\,347\,391 \times 10^{19}$	33	4	3	$0.510\,655\,119\,774\,360 \times 10^{17}$
14	−5	14	$-0.103\,977\,184\,454\,767 \times 10^{29}$	34	4	10	$-0.468\,138\,358\,908\,732 \times 10^{32}$
15	−4	−12	$0.119\,563\,135\,540\,666 \times 10^{-47}$	35	5	2	$-0.760\,667\,491\,183\,279 \times 10^{16}$
16	−4	−10	$-0.436\,677\,034\,051\,655 \times 10^{-41}$	36	8	−12	$-0.417\,247\,986\,986\,821 \times 10^{-18}$
17	−4	−6	$0.926\,990\,036\,530\,639 \times 10^{-29}$	37	10	−2	$0.312\,545\,677\,756\,104 \times 10^{14}$
18	−4	10	$0.587\,793\,105\,620\,748 \times 10^{21}$	38	12	−3	$-0.100\,375\,333\,864\,186 \times 10^{15}$
19	−3	−3	$0.280\,375\,725\,094\,731 \times 10^{-17}$	39	14	1	$0.247\,761\,392\,329\,058 \times 10^{27}$
20	−3	10	$-0.192\,359\,972\,440\,634 \times 10^{23}$				

Table A2.3. Coefficients and exponents of the auxiliary equation $v_{3w}(p,T)$ for subregion 3w

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	8	$-0.586\,219\,133\,817\,016 \times 10^{-7}$	19	−1	−8	$0.237\,416\,732\,616\,644 \times 10^{-26}$
2	−12	14	$-0.894\,460\,355\,005\,526 \times 10^{11}$	20	−1	−4	$0.271\,700\,235\,739\,893 \times 10^{-14}$
3	−10	−1	$0.531\,168\,037\,519\,774 \times 10^{-30}$	21	−1	1	$-0.907\,886\,213\,483\,600 \times 10^{2}$
4	−10	8	$0.109\,892\,402\,329\,239$	22	0	−12	$-0.171\,242\,509\,570\,207 \times 10^{-36}$
5	−8	6	$-0.575\,368\,389\,425\,212 \times 10^{-1}$	23	0	1	$0.156\,792\,067\,854\,621 \times 10^{3}$
6	−8	8	$0.228\,276\,853\,990\,249 \times 10^{5}$	24	1	−1	$0.923\,261\,357\,901\,470$
7	−8	14	$-0.158\,548\,609\,655\,002 \times 10^{19}$	25	2	−1	$-0.597\,865\,988\,422\,577 \times 10^{1}$
8	−6	−4	$0.329\,865\,748\,576\,503 \times 10^{-27}$	26	2	2	$0.321\,988\,767\,636\,389 \times 10^{7}$
9	−6	−3	$-0.634\,987\,981\,190\,669 \times 10^{-24}$	27	3	−12	$-0.399\,441\,390\,042\,203 \times 10^{-29}$
10	−6	2	$0.615\,762\,068\,640\,611 \times 10^{-8}$	28	3	−5	$0.493\,429\,086\,046\,981 \times 10^{-7}$
11	−6	8	$-0.961\,109\,240\,985\,747 \times 10^{8}$	29	5	−10	$0.812\,036\,983\,370\,565 \times 10^{-19}$
12	−5	−10	$-0.406\,274\,286\,652\,625 \times 10^{-44}$	30	5	−8	$-0.207\,610\,284\,654\,137 \times 10^{-11}$
13	−4	−1	$-0.471\,103\,725\,498\,077 \times 10^{-12}$	31	5	−6	$-0.340\,821\,291\,419\,719 \times 10^{-6}$
14	−4	3	$0.725\,937\,724\,828\,145$	32	8	−12	$0.542\,000\,573\,372\,233 \times 10^{-17}$
15	−3	−10	$0.187\,768\,525\,763\,682 \times 10^{-38}$	33	8	−10	$-0.856\,711\,586\,510\,214 \times 10^{-12}$
16	−3	3	$-0.103\,308\,436\,323\,771 \times 10^{4}$	34	10	−12	$0.266\,170\,454\,405\,981 \times 10^{-13}$
17	−2	1	$-0.662\,552\,816\,342\,168 \times 10^{-1}$	35	10	−8	$0.858\,133\,791\,857\,099 \times 10^{-5}$
18	−2	2	$0.579\,514\,041\,765\,710 \times 10^{3}$				

Table A2.4. Coefficients and exponents of the auxiliary equation $v_{3x}(p,T)$ for subregion 3x

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−8	14	$0.377\,373\,741\,298\,151 \times 10^{19}$	19	4	3	$0.397\,949\,001\,553\,184 \times 10^{14}$
2	−6	10	$-0.507\,100\,883\,722\,913 \times 10^{13}$	20	5	−6	$0.100\,824\,008\,584\,757 \times 10^{-6}$
3	−5	10	$-0.103\,363\,225\,598\,860 \times 10^{16}$	21	5	−2	$0.162\,234\,569\,738\,433 \times 10^{5}$
4	−4	1	$0.184\,790\,814\,320\,773 \times 10^{-5}$	22	5	1	$-0.432\,355\,225\,319\,745 \times 10^{11}$
5	−4	2	$-0.924\,729\,378\,390\,945 \times 10^{-3}$	23	6	1	$-0.592\,874\,245\,598\,610 \times 10^{12}$
6	−4	14	$-0.425\,999\,562\,292\,738 \times 10^{24}$	24	8	−6	$0.133\,061\,647\,281\,106 \times 10^{1}$
7	−3	−2	$-0.462\,307\,771\,873\,973 \times 10^{-12}$	25	8	−3	$0.157\,338\,197\,797\,544 \times 10^{7}$
8	−3	12	$0.107\,319\,065\,855\,767 \times 10^{22}$	26	8	1	$0.258\,189\,614\,270\,853 \times 10^{14}$
9	−1	5	$0.648\,662\,492\,280\,682 \times 10^{11}$	27	8	8	$0.262\,413\,209\,706\,358 \times 10^{25}$
10	0	0	$0.244\,200\,600\,688\,281 \times 10^{1}$	28	10	−8	$-0.920\,011\,937\,431\,142 \times 10^{-1}$
11	0	4	$-0.851\,535\,733\,484\,258 \times 10^{10}$	29	12	−10	$0.220\,213\,765\,905\,426 \times 10^{-2}$
12	0	10	$0.169\,894\,481\,433\,592 \times 10^{22}$	30	12	−8	$-0.110\,433\,759\,109\,547 \times 10^{2}$
13	1	−10	$0.215\,780\,222\,509\,020 \times 10^{-26}$	31	12	−5	$0.847\,004\,870\,612\,087 \times 10^{7}$
14	1	−1	$-0.320\,850\,551\,367\,334$	32	12	−4	$-0.592\,910\,695\,762\,536 \times 10^{9}$
15	2	6	$-0.382\,642\,448\,458\,610 \times 10^{17}$	33	14	−12	$-0.183\,027\,173\,269\,660 \times 10^{-4}$
16	3	−12	$-0.275\,386\,077\,674\,421 \times 10^{-28}$	34	14	−10	$0.181\,339\,603\,516\,302$
17	3	0	$-0.563\,199\,253\,391\,666 \times 10^{6}$	35	14	−8	$-0.119\,228\,759\,669\,889 \times 10^{4}$
18	3	8	$-0.326\,068\,646\,279\,314 \times 10^{21}$	36	14	−6	$0.430\,867\,658\,061\,468 \times 10^{7}$

Table A2.5. Coefficients and exponents of the auxiliary equation $v_{3y}(p,T)$ for subregion 3y

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	−3	$-0.525\,597\,995\,024\,633 \times 10^{-9}$	11	3	4	$0.705\,106\,224\,399\,834 \times 10^{21}$
2	0	1	$0.583\,441\,305\,228\,407 \times 10^{4}$	12	3	8	$-0.266\,713\,136\,106\,469 \times 10^{31}$
3	0	5	$-0.134\,778\,968\,457\,925 \times 10^{17}$	13	4	−6	$-0.145\,370\,512\,554\,562 \times 10^{-7}$
4	0	8	$0.118\,973\,500\,934\,212 \times 10^{26}$	14	4	6	$0.149\,333\,917\,053\,130 \times 10^{28}$
5	1	8	$-0.159\,096\,490\,904\,708 \times 10^{27}$	15	5	−2	$-0.149\,795\,620\,287\,641 \times 10^{8}$
6	2	−4	$-0.315\,839\,902\,302\,021 \times 10^{-6}$	16	5	1	$-0.381\,881\,906\,271\,100 \times 10^{16}$
7	2	−1	$0.496\,212\,197\,158\,239 \times 10^{3}$	17	8	−8	$0.724\,660\,165\,585\,797 \times 10^{-4}$
8	2	4	$0.327\,777\,227\,273\,171 \times 10^{19}$	18	8	−2	$-0.937\,808\,169\,550\,193 \times 10^{14}$
9	2	5	$-0.527\,114\,657\,850\,696 \times 10^{22}$	19	10	−5	$0.514\,411\,468\,376\,383 \times 10^{10}$
10	3	−8	$0.210\,017\,506\,281\,863 \times 10^{-16}$	20	12	−8	$-0.828\,198\,594\,040\,141 \times 10^{5}$

Table A2.6. Coefficients and exponents of the auxiliary equation $v_{3z}(p,T)$ for subregion 3z

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−8	3	$0.244\,007\,892\,290\,650 \times 10^{-10}$	13	0	3	$0.328\,380\,587\,890\,663 \times 10^{12}$
2	−6	6	$-0.463\,057\,430\,331\,242 \times 10^{7}$	14	1	1	$-0.625\,004\,791\,171\,543 \times 10^{8}$
3	−5	6	$0.728\,803\,274\,777\,712 \times 10^{10}$	15	2	6	$0.803\,197\,957\,462\,023 \times 10^{21}$
4	−5	8	$0.327\,776\,302\,858\,856 \times 10^{16}$	16	3	−6	$-0.204\,397\,011\,338\,353 \times 10^{-10}$
5	−4	5	$-0.110\,598\,170\,118\,409 \times 10^{10}$	17	3	−2	$-0.378\,391\,047\,055\,938 \times 10^{4}$
6	−4	6	$-0.323\,899\,915\,729\,957 \times 10^{13}$	18	6	−6	$0.972\,876\,545\,938\,620 \times 10^{-2}$
7	−4	8	$0.923\,814\,007\,023\,245 \times 10^{16}$	19	6	−5	$0.154\,355\,721\,681\,459 \times 10^{2}$
8	−3	−2	$0.842\,250\,080\,413\,712 \times 10^{-12}$	20	6	−4	$-0.373\,962\,862\,928\,643 \times 10^{4}$
9	−3	5	$0.663\,221\,436\,245\,506 \times 10^{12}$	21	6	−1	$-0.682\,859\,011\,374\,572 \times 10^{11}$
10	−3	6	$-0.167\,170\,186\,672\,139 \times 10^{15}$	22	8	−8	$-0.248\,488\,015\,614\,543 \times 10^{-3}$
11	−2	2	$0.253\,749\,358\,701\,391 \times 10^{4}$	23	8	−4	$0.394\,536\,049\,497\,068 \times 10^{7}$
12	−1	−6	$-0.819\,731\,559\,610\,523 \times 10^{-20}$				